2.1–Use Inductive Reasoning

- Conjecture –

- Inductive reasoning –

- Counterexample –

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**Example 1** Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Figure 1

![Figure 1](image1)

Figure 2

![Figure 2](image2)

Figure 3

![Figure 3](image3)

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1. Sketch the fifth figure in the pattern in Example 1.

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**Example 3** Make a conjecture

Given five noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Make a table and look for a pattern. Notice the pattern in how the number of connections ________. You can use the pattern to make a conjecture.

<table>
<thead>
<tr>
<th>Number of points</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Picture</td>
<td>.</td>
<td>-</td>
<td>△</td>
<td>□</td>
<td>☐</td>
</tr>
<tr>
<td>Number of connections</td>
<td>__</td>
<td>___</td>
<td>___</td>
<td>___</td>
<td>___</td>
</tr>
</tbody>
</table>

___ + ___ + ___ + ___ + ___

Conjecture You can connect five noncollinear points ________, or ____ different ways.
2. Describe the pattern in the numbers 1, 2.5, 4, 5.5, . . . and write the next three numbers in the pattern.

Example 4  Make and test a conjecture

Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of any three consecutive odd numbers.

Example 5  Find a counterexample

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student’s conjecture.

Conjecture The difference of any two numbers is always smaller than the larger number.

Example 6  Making conjectures from data displays

The scatter plot shows the average salary of players in the National Football League (NFL) since 1999. Make a conjecture based on the graph.

5. Find a counterexample to show that the following conjecture is false.

Conjecture The quotient of two numbers is always smaller than the dividend.
2.2 – Analyze Conditional Statements

- Conditional statement
- If-then form
- Negation

- Conditional statement
  - Converse
  - Inverse
  - Contrapositive

- Equivalent Statements
- Biconditional statements

- Perpendicular Lines (Definition)

**Perpendicular Lines**

*Definition*: Perpendicular lines are two lines that intersect to form a right angle. The symbol used for perpendicular lines is \( \perp \). 4 right angles are formed.

In this figure line \( m \) is perpendicular to line \( n \).

With symbols we denote, \( m \perp n \)
Example 1  **Rewriting in If-Then Form**

Rewrite the conditional statement in *if-then* form.

a. Three points are coplanar if they lie on the same plane.

b. Water freezes at temperatures below $32^\circ F$.

c. An even number is divisible by 2.

Example 2  **Writing an Inverse, Converse, and Contrapositive**

Write the (a) inverse, (b) converse, and (c) contrapositive of the following statement.

If the sun is shining, then we are not watching TV.

**Solution**

a. Inverse: ____________________________

b. Converse: __________________________

c. Contrapositive: __________________________

Example 1  **Using Definitions**

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

a. $\angle KLJ$ and $\angle KJL$ are complementary.

b. $\overrightarrow{KL}$ and $\overrightarrow{LJ}$ are perpendicular.

c. $\angle MKJ$ is a right angle.

Example 2  **Rewriting a Biconditional Statement**

Rewrite the following biconditional statement as a conditional statement and its converse.

An angle is a straight angle if and only if its measure is $180^\circ$.

Example 3  **Analyzing a Biconditional Statement**

Consider the following statement: $x = 2$ if and only if $3x + 5x = 10x - 2x$.

a. Is this a biconditional statement?  b. Is the statement true?
2.3 – Apply Deductive Reasoning

Deductive reasoning –

**LAW OF DETACHMENT**
If $p \implies q$ is a true conditional statement and $p$ is true, then __________.

**LAW OF SYLLOGISM**
If $p \implies q$ and $q \implies r$ are true conditional statements, then __________.

**Example 3** Using the Law of Detachment

State whether the argument is valid.

a. If Roger gets a part-time job, then he will buy a new bicycle. Roger buys a new bicycle. So, Roger got a part-time job.

b. If two angles are vertical angles, then they are congruent. $\angle 1$ and $\angle 2$ are vertical angles. So, $\angle 1$ and $\angle 2$ are congruent.

5. State whether the following argument is valid. If two adjacent angles form a straight angle, then the angles are supplementary. $\angle 1$ and $\angle 2$ are supplementary. So, you can conclude that $\angle 1$ and $\angle 2$ are adjacent.

**Example 4** Using the Law of Syllogism

Write some conditional statements that can be made from the following true statements using the Law of Syllogism.

1. If a cat is the largest of all cats, then it can weigh 650 pounds.
2. If a cat lives in a pride, then it is a lion.
3. If a cat weighs 650 pounds, then it is a tiger.
4. If a cat is a tiger, then it hunts alone.
5. If a cat is a lion, then it can weigh 400 pounds.
2.4 – Use Postulates and Diagrams

**POINT, LINE, AND PLANE POSTULATES**

Postulate 5  Through any two points there exists exactly one _____.

Postulate 6  A line contains at least two _______.

Postulate 7  If two lines intersect, then their intersection is _______.

Postulate 8  Through any three __________ points there exists exactly one plane.

Postulate 9  A plane contains at least three __________ points.

Postulate 10 If two points lie in a plane, then the line containing them __________.

Postulate 11 If two planes intersect, then their intersection is a _______.

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**Example 1**  *Identify a postulate illustrated by a diagram*

State the postulate illustrated by the diagram.

If \( A \cdot B \cdot C \cdot \) then \( A \cdot B \cdot C \cdot \)
Example 2  *Identify postulates from a diagram*

Use the diagram to write examples of Postulates 9 and 11.

**Postulate 9** Plane contains at least three noncollinear points, ________.

**Postulate 11** The intersection of plane $P$ and plane $Q$ is ______.

**Checkpoint** Use the diagram in Example 2 to complete the following exercises.

1. Which postulate allows you to say that the intersection of line $a$ and line $b$ is a point?

2. Write examples of Postulates 5 and 6.

**CONCEPT SUMMARY: INTERPRETING A DIAGRAM**

When you interpret a diagram, you can only assume information about size or measure if it is marked.

**YOU CANNOT ASSUME**

- All points shown are ________.
- $\angle AHB$ and ________ are a linear pair.
- $\angle AHF$ and ________ are vertical angles.
- $A$, $H$, $J$, and $D$ are ________.
- $\overline{AD}$ and $\overline{BF}$ intersect at ______.

**YOU CANNOT ASSUME**

- $G$, $F$, and $E$ are collinear.
- $\overline{BF}$ and $\overline{CE}$ intersect.
- $\overline{BF}$ and $\overline{CE}$ do not intersect.
- $\angle BHA \equiv \angle CJA$
- $\overline{AD} \perp \overline{BF}$ or $m\angle AHB = 90^\circ$
2.5 – Reason Using Properties from Algebra

**ALGEBRAIC PROPERTIES OF EQUALITY**

Let $a$, $b$, and $c$ be real numbers.

- **Addition Property**  
  If $a = b$, then \[ \text{______________} \].

- **Subtraction Property**  
  If $a = b$, then \[ \text{______________} \].

- **Multiplication Property**  
  If $a = b$, then \[ \text{______________} \].

- **Division Property**  
  If $a = b$ and $c \neq 0$, then \[ \text{______________} \].

- **Reflexive Property**  
  For any real number $a$, \[ \text{______} \].

- **Symmetric Property**  
  If $a = b$, then \[ \text{______} \].

- **Transitive Property**  
  If $a = b$ and $b = c$, then \[ \text{______} \].

- **Substitution Property**  
  If $a = b$, then \[ \text{___________________} \].

---

**Example 1**  
**Writing Reasons**

Solve $-2x + 1 = 56 - 3x$ and write a reason for each step.

\[-2x + 1 = 56 - 3x \quad \text{Given}\]

\[\quad + 1 = 56 \quad \text{______________________________}\]

\[x = \__ \quad \text{______________________________}\]

**Checkpoint**  
Solve the equation. Write a reason for each step.

1. $12x - 3(x + 7) = 8x$
**PROPERTIES OF EQUALITY**

<table>
<thead>
<tr>
<th>Segment Length</th>
<th>Angle Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>For any segment AB, for any angle A,</td>
</tr>
<tr>
<td>Symmetric</td>
<td>If AB = CD, then If m∠A = m∠B, then</td>
</tr>
<tr>
<td>Transitive</td>
<td>If AB = CD and CD = EF, If m∠A = m∠B and m∠B = m∠C, then</td>
</tr>
</tbody>
</table>

**Example 3** Using Properties of Measure

Use the information at the right to find m∠1.

m∠1 + m∠2 + m∠3 + m∠4 = 360°

m∠2 + m∠3 = m∠4

m∠1 = m∠4

**Solution**

\[
m∠1 + m∠2 + m∠3 + m∠4 = \______ \\
m∠2 + m∠3 = \______ \\
m∠1 = \______ \\
\______ + \______ + \______ = 360° \\
3(\______ ) = 360° \\
\______ = \______ \\
m∠1 = \______
\]

**Checkpoint** Complete the following exercise.

2. In the diagram at the right, B is the midpoint of AC and C is the midpoint of BD. Show that AB = CD.
2.6 – Prove Statements about Segments and Angles

- Proof –

THEOREM 2.2 PROPERTIES OF ANGLE CONGRUENCE

Angle congruence is reflexive, symmetric, and transitive.

- Reflexive
  For any angle \( \angle A \),

- Symmetric
  If \( \angle A \equiv \angle B \), then

- Transitive
  If \( \angle A \equiv \angle B \) and \( \angle B \equiv \angle C \), then

THEOREM 2.1 PROPERTIES OF SEGMENT CONGRUENCE

- Reflexive
  For any segment \( AB \),

- Symmetric
  If \( AB \equiv CD \), then

- Transitive
  If \( AB \equiv CD \), and \( CD \equiv EF \), then

Example 1: Transitive Property of Segment Congruence

You can prove the Transitive Property of Segment Congruence as follows.

Given: \( JK \equiv MN \), \( MN \equiv PQ \)
Prove: \( JK \equiv PQ \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( JK \equiv MN ), ( MN \equiv PQ )</td>
<td>1.</td>
</tr>
<tr>
<td>2. ( JK = MN ), ( MN = PQ )</td>
<td>2.</td>
</tr>
<tr>
<td>3.</td>
<td>3. Transitive property of equality</td>
</tr>
<tr>
<td>4. ( JK \equiv PQ )</td>
<td>4. Definition of congruent segments</td>
</tr>
</tbody>
</table>
Example 2  Using Congruence

Use the diagram and the given information to complete the proof.

Given: \( PQ \cong RS, PQ \cong QR, PS \cong RS \)
Prove: \( PS \cong QR \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( PQ \cong RS )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( PQ \cong QR )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( RS \cong QR )</td>
<td>3. Transitive Property of Congruence</td>
</tr>
<tr>
<td>4. ( PS \cong RS )</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( PS \cong QR )</td>
<td>5. Transitive Property of Congruence</td>
</tr>
</tbody>
</table>

Example 3  Using Segment Relationships

In the diagram, \( AC = CE \) and \( AB = DE \).
Show that \( C \) is the midpoint of \( BD \).

Solution

Given: _____________________________

Prove: _____________________________

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AC = CE )</td>
<td>1.</td>
</tr>
<tr>
<td>2. ( AB + BC = AC )</td>
<td>2.</td>
</tr>
<tr>
<td>3.</td>
<td>3. Transitive Property of Equality</td>
</tr>
<tr>
<td>4. ( CD + DE = CE )</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5. Transitive Property of Equality</td>
</tr>
<tr>
<td>6. ( AB = DE )</td>
<td>6.</td>
</tr>
<tr>
<td>7. ( AB + BC = CD + AB )</td>
<td>7.</td>
</tr>
<tr>
<td>8.</td>
<td>8. Subtraction Property of Equality</td>
</tr>
<tr>
<td>9.</td>
<td>9. Definition of congruent segments</td>
</tr>
<tr>
<td>10. ( C ) is the midpoint of ( BD ).</td>
<td>10.</td>
</tr>
</tbody>
</table>

Example 1  Using the Transitive Property

In the diagram at the right, \( \angle 1 \cong \angle 5, \)
\( \angle 5 \cong \angle 3, \) and \( m\angle 1 = 103^\circ \).
What is the measure of \( \angle 3 \)? Explain your reasoning.
2.7 – Prove Angle Pair Relationships

THEOREM 2.3 RIGHT ANGLE CONGRUENCE THEOREM
All right angles are \( \underline{\text{________}} \).

THEOREM 2.4 CONGRUENT SUPPLEMENTS THEOREM
If two angles are supplementary to the same angle (or to congruent angles), then they are \( \underline{\text{________}} \).
If \( m\angle 1 + m\angle 2 = \underline{\text{_____}} \) and \( m\angle 2 + m\angle 3 = \underline{\text{_____}} \), then \( \underline{\text{______}} \).

THEOREM 2.5 CONGRUENT COMPLEMENTS THEOREM
If two angles are complementary to the same angle (or to congruent angles), then the two angles are \( \underline{\text{________}} \).
If \( m\angle 4 + m\angle 5 = \underline{\text{_____}} \) and \( m\angle 5 + m\angle 6 = \underline{\text{_____}} \), then \( \underline{\text{______}} \).

POSTULATE 12 LINEAR PAIR POSTULATE
If two angles form a linear pair, then they are \( \underline{\text{________}} \).
\( m\angle 1 + m\angle 2 = \underline{\text{_____}} \)

THEOREM 2.6 VERTICAL ANGLES THEOREM
Vertical angles are \( \underline{\text{________}} \).
\( \angle 1 \cong \underline{\text{_____}} \) and \( \underline{\text{_____}} \cong \angle 4 \)
Example 2  Proving Theorem 2.5

Given: $\angle 1$ and $\angle 2$ are complements, $\angle 3$ and $\angle 4$ are complements, $\angle 2 \cong \angle 4$

Prove: $\angle 1 \cong \angle 3$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1$ and $\angle 2$ are complements,</td>
<td>1.</td>
</tr>
<tr>
<td>$\angle 3$ and $\angle 4$ are complements,</td>
<td></td>
</tr>
<tr>
<td>$\angle 2 \cong \angle 4$</td>
<td></td>
</tr>
<tr>
<td>2. $m\angle 1 + m\angle 2 = 90^\circ$</td>
<td>2.</td>
</tr>
<tr>
<td>$m\angle 3 + m\angle 4 = 90^\circ$</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>3. Transitive property of</td>
</tr>
<tr>
<td></td>
<td>equality</td>
</tr>
<tr>
<td>4. $m\angle 2 = m\angle 4$</td>
<td>4.</td>
</tr>
<tr>
<td>5.</td>
<td>5. Substitution property of</td>
</tr>
<tr>
<td></td>
<td>equality</td>
</tr>
<tr>
<td>6.</td>
<td>6. Subtraction property of</td>
</tr>
<tr>
<td></td>
<td>equality</td>
</tr>
<tr>
<td>7. $\angle 1 \cong \angle 3$</td>
<td>7.</td>
</tr>
</tbody>
</table>

Example 3  Using Linear Pairs and Vertical Angles

In the diagram, $\angle 3$ is a right angle and $m\angle 5 = 57^\circ$. Find the measures of $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$.

2. Find $m\angle 1$ and $m\angle 2$.

3. Find the measure of each angle.