Theorems

2.1	**Properties of Segment Congruence** - Segment congruence is reflexive, symmetric, and transitive.
2.2	**Properties of Angle Congruence** - Angle congruence is reflexive, symmetric, and transitive.
2.3	**Right Angles Congruence Theorem** - All right angles are congruent.
2.4	**Congruent Supplements Theorem** - If two angles are supplementary to the same angle (or to congruent angles), then the two angles are congruent.
2.5	**Congruent Complements Theorem** - If two angles are complementary to the same angle (or to congruent angles), then the two angles are congruent.
2.6	**Vertical Angles Congruence Theorem** - Vertical angles are congruent.

3.1	**Alternate Interior Angles Theorem** - If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.
3.2	**Alternate Exterior Angles Theorem** - If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.
3.3	**Consecutive Interior Angles Theorem** - If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are

	supplementary.
3.4	**Alternate Interior Angles Converse** - If two lines are cut by a transversal so the alternate interior angles are congruent, then the two lines are parallel.
3.5	**Alternate Exterior Angles Converse** - If two lines are cut by a transversal so the alternate exterior angles are congruent, then the two lines are parallel.
3.6	**Consecutive Interior Angles Converse** - If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the two lines are parallel.
3.7	**Transitive Property of Parallel Lines** - If two lines are parallel to the same line, then they are parallel to each other.
3.8	If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.
3.9	If two lines are perpendicular, then they intersect to form four right angles.
3.10	If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.
3.11	**Perpendicular Transversal Theorem** - If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.
3.12	**Lines Perpendicular to a Transversal Theorem** - In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

4.1	**Triangle Sum Theorem** - The sum of the measures of the interior angles of a triangle is 180.
4.2	**Exterior Angle Theorem** - The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.
4.3	**Third Angles Theorem** - If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.
4.4	**Properties of Triangle Congruence** - Triangle congruence is reflexive, symmetric, and transitive.
4.5	**Hypotenuse-Leg (HL) Congruence Theorem** - If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and leg of a second right triangle, then the two triangles are congruent.
4.6	**Angle-Angle-Side (AAS) Congruence Theorem** - If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of second triangle, then the two triangles are congruent.
4.7	**Base Angles Theorem** - If two sides of a triangle are congruent, then the angles opposite them are congruent.
cor.	If a triangle is equilateral, then it is equiangular.
4.8	**Converse of the Base Angles Theorem** - If two angles of a triangle are congruent, then the sides opposite them are congruent.
cor.	If a triangle is equiangular, then it is equilateral.

5.1	**Midsegment Theorem** - The segment connecting the midpoint of two sides of a triangle is parallel to the third side and is half as long as that side.
5.2	**Perpendicular Bisector Theorem** - If a point is on a perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.
5.3	**Converse of the Perpendicular Bisector Theorem** - If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.
5.4	**Concurrency of Perpendicular Bisectors Theorem** - The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.
5.5	**Angle Bisector Theorem** - If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.
5.6	**Converse of the Angle Bisector Theorem** - If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.
5.7	**Concurrency of Angle Bisectors of a Triangle** - The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.
5.8	**Concurrency of Medians of a Triangle** - The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

5.9	**Concurrency of Altitudes of a Triangle** - The lines containing the altitudes of a triangle are concurrent.
5.10	If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.
5.11	If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.
5.12	**Triangle Inequality Theorem** - The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
5.13	**Hinge Theorem** - If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.
5.14	**Converse of the Hinge Theorem** - If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.

6.1	If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.
6.2	**Side-Side-Side (SSS) Similarity Theorem -** If the corresponding side lengths of two triangles are proportional, the the triangles are similar.

6.3	**Side-Angle-Side (SAS) Similarity Theorem -** If an angle of one triangle is congruent to an angle of a second triangle and the lengths or the sides including these angles are proportional, the the triangles are similar.
6.4	**Triangle Proportionality Theorem** - If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.
6.5	**Converse of the Triangle Proportionality Theorem** - If a line divides two sides of a triangle proportionally, then it is parallel to the third side.
6.6	If three parallel lines intersect two transversals, then they divide the transversals proportionally.
6.7	If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

7.1	Pythagorean Theorem - In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.
7.2	Converse of the Pythagorean Theorem - If the square of the length of the longest side of a traingle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.
7.3	If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the

	triangle is an acute triangle.
7.4	If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is an obtuse triangle.
7.5	If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.
7.6	Geometric Mean (Altitude) Theorem - In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of the altitude is the geometric mean of the lengths of the two segments.
7.7	Geometric Mean (Leg) Theorem - In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.
7.8	45°-45°-90° Triangle Theorem - In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.
7.9	30°-60°-90° Triangle Theorem - In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

8.1	**Polygon Interior Angles Theorem** The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \cdot 180°$. *(p. 507)* **Corollary** The sum of the measures of the interior angles of a quadrilateral is $360°$. *(p. 507)*
8.2	**Polygon Exterior Angles Theorem** The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is $360°$. *(p. 509)*
8.3	If a quadrilateral is a parallelogram, then its opposite sides are congruent. *(p. 515)*
8.4	If a quadrilateral is a parallelogram, then its opposite angles are congruent. *(p. 515)*
8.5	If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. *(p. 516)*
8.6	If a quadrilateral is a parallelogram, then its diagonals bisect each other. *(p. 517)*
8.7	If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. *(p. 522)*
8.8	If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. *(p. 522)*
8.9	If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram. *(p. 523)*

8.10	If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. *(p. 523)*
	Rhombus Corollary A quadrilateral is a rhombus if and only if it has four congruent sides. *(p. 533)* **Rectangle Corollary** A quadrilateral is a rectangle if and only if it has four right angles. *(p. 533)* **Square Corollary** A quadrilateral is a square if and only if it is a rhombus and a rectangle. *(p. 533)*
8.11	A parallelogram is a rhombus if and only if its diagonals are perpendicular. *(p. 535)*
8.12	A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles. *(p. 535)*
8.13	A parallelogram is a rectangle if and only if its diagonals are congruent. *(p. 535)*
8.14	If a trapezoid is isosceles, then both pairs of base angles are congruent. *(p. 543)*
8.15	If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid. *(p. 543)*
8.16	A trapezoid is isosceles if and only if its diagonals are congruent. *(p. 543)*
8.17	**Midsegment Theorem for Trapezoids** The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases. *(p. 544)*

8.18	If a quadrilateral is a kite, then its diagonals are perpendicular. *(p. 545)*
8.19	If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent. *(p. 545)*

9.1	**Translation Theorem** A translation is an isometry. *(p. 573)*
9.2	**Reflection Theorem** A reflection is an isometry. *(p. 591)*
9.3	**Rotation Theorem** A rotation is an isometry. *(p. 601)*
9.4	**Composition Theorem** The composition of two (or more) isometries is an isometry. *(p. 609)*
9.5	**Reflections in Parallel Lines** If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation. If P'' is the image of P, then: (1) $\overline{PP'}$ is perpendicular to k and m, and (2) $PP'' = 2d$, where d is the distance between k and m. *(p. 609)*
9.6	**Reflections in Intersecting Lines** If lines k and m intersect at point P, then a reflection in k followed by a reflection in m is the same as a rotation about point P. The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by k and m. *(p. 610)*

10.1	In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle. *(p. 653)*
10.2	Tangent segments from a common external point are congruent. *(p. 654)*
10.3	In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. *(p. 664)*
10.4	If one chord is a perpendicular bisector of another chord, then the first chord is a diameter. *(p. 665)*
10.5	If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc. *(p. 665)*
10.6	In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center. *(p. 666)*
10.7	**Measure of an Inscribed Angle Theorem** The measure of an inscribed angle is one half the measure of its intercepted arc. *(p. 672)*
10.8	If two inscribed angles of a circle intercept the same arc, then the angles are congruent. *(p. 673)*
10.9	If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle. *(p. 674)*
10.10	A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary. *(p. 675)*

10.11	If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc. *(p. 680)*
10.12	**Angles Inside the Circle** If two chords intersect inside a circle, then the measure of each angle is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle. *(p. 681)*
10.13	**Angles Outside the Circle** If a tangent and a secant, two tangents, or two secants intersect outside a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs. *(p. 681)*
10.14	**Segments of Chords Theorem** If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. *(p. 689)*
10.15	**Segments of Secants Theorem** If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment. *(p. 690)*
10.16	**Segments of Secants and Tangents Theorem** If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment. *(p. 691)*

11.1	**Area of a Rectangle** The area of a rectangle is the product of its base and height. $A = bh$ *(p. 720)*
11.2	**Area of a Parallelogram** The area of a parallelogram is the product of a base and its corresponding height. $A = bh$ *(p. 721)*

11.3	**Area of a Triangle** The area of a triangle is one half the product of a base and its corresponding height. $A = \frac{1}{2}bh$ *(p. 721)*
11.4	**Area of a Trapezoid** The area of a trapezoid is one half the product of the height and the sum of the lengths of the bases. $A = \frac{1}{2}h(b_1 + b_2)$ *(p. 730)*
11.5	**Area of a Rhombus** The area of a rhombus is one half the product of the lengths of its diagonals. $A = \frac{1}{2}d_1d_2$ *(p. 731)*
11.6	**Area of a Kite** The area of a kite is one half the product of the lengths of its diagonals. $A = \frac{1}{2}d_1d_2$ *(p. 731)*
11.7	**Areas of Similar Polygons** If two polygons are similar with the lengths of corresponding sides in the ratio of $a : b$, then the ratio of their areas is $a^2 : b^2$. *(p. 737)*
11.8	**Circumference of a Circle** The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle. *(p. 746)*
	Arc Length Corollary In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°. $\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360°}$, or Arc length of $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r$ *(p. 747)*
11.9	**Area of a Circle** The area of a circle is π times the square of the radius. $A = \pi r^2$ *(p. 755)*
11.10	**Area of a Sector** The ratio of the area A of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°. $\frac{A}{\pi r^2} = \frac{m\widehat{AB}}{360°}$, or $A = \frac{m\widehat{AB}}{360°} \cdot \pi r^2$ *(p. 756)*

11.11	**Area of a Regular Polygon** The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P, so $A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \cdot ns$. *(p. 763)*

12.1	**Euler's Theorem** The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$. *(p. 795)*
12.2	**Surface Area of a Right Prism** The surface area S of a right prism is $S = 2B + Ph = aP + Ph$, where a is the apothem of the base, B is the area of a base, P is the perimeter of a base, and h is the height. *(p. 804)*
12.3	**Surface Area of a Right Cylinder** The surface area S of a right cylinder is $S = 2B + Ch = 2\pi r^2 + 2\pi rh$, where B is the area of a base, C is the circumference of a base, r is the radius of a base, and h is the height. *(p. 805)*
12.4	**Surface Area of a Regular Pyramid** The surface area S of a regular pyramid is $S = B + \frac{1}{2}P\ell$, where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height. *(p. 811)*
12.5	**Surface Area of a Right Cone** The surface area S of a right cone is $S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell$, where B is the area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height. *(p. 812)*
12.6	**Volume of a Prism** The volume V of a prism is $V = Bh$, where B is the area of a base and h is the height. *(p. 820)*
12.7	**Volume of a Cylinder** The volume V of a cylinder is $V = Bh = \pi r^2h$, where B is the area of a base, h is the height, and r is the radius of a base. *(p. 820)*

12.8	**Cavalieri's Principle** If two solids have the same height and the same cross-sectional area at every level, then they have the same volume. *(p. 821)*
12.9	**Volume of a Pyramid** The volume V of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height. *(p. 829)*
12.10	**Volume of a Cone** The volume V of a cone is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$, where B is the area of the base, h is the height, and r is the radius of the base. *(p. 829)*
12.11	**Surface Area of a Sphere** The surface area S of a sphere with radius r is $S = 4\pi r^2$. *(p. 838)*
12.12	**Volume of a Sphere** The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. *(p. 840)*
12.13	**Similar Solids Theorem** If two similar solids have a scale factor of $a : b$, then corresponding areas have a ratio of $a^2 : b^2$, and corresponding volumes have a ratio of $a^3 : b^3$. *(p. 848)*

Postulates

1 Ruler Postulate The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the coordinate of the point. The distance between points A and B, written as AB, is the absolute value of the difference between the coordinates of A and B. *(p. 9)*

2 Segment Addition Postulate If B is between A and C, then $AB + BC = AC$. If $AB + BC = AC$, then B is between A and C. *(p. 10)*

3 Protractor Postulate Consider $\overrightarrow{OB}$ and a point A on one side of $\overrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180. The measure of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{OA}$ and $\overrightarrow{OB}$. *(p. 24)*

4 Angle Addition Postulate If P is in the interior of $\angle RST$, then $m\angle RST = m\angle RSP + m\angle PST$. *(p. 25)*

5 Through any two points there exists exactly one line. *(p. 96)*

6 A line contains at least two points. *(p. 96)*

7 If two lines intersect, then their intersection is exactly one point. *(p. 96)*

8 Through any three noncollinear points there exists exactly one plane. *(p. 96)*

9 A plane contains at least three noncollinear points. *(p. 96)*

10 If two points lie in a plane, then the line containing them lies in the plane. *(p. 96)*

11 If two planes intersect, then their intersection is a line. *(p. 96)*

12 Linear Pair Postulate If two angles form a linear pair, then they are supplementary. *(p. 126)*

13 Parallel Postulate If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line. *(p. 148)*

14 Perpendicular Postulate If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line. *(p. 148)*

15 Corresponding Angles Postulate If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent. *(p. 154)*

16 Corresponding Angles Converse If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel. *(p. 161)*

17 Slopes of Parallel Lines In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel. *(p. 172)*

18 Slopes of Perpendicular Lines In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1. Horizontal lines are perpendicular to vertical lines. *(p. 172)*

19 Side-Side-Side (SSS) Congruence Postulate If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent. *(p. 234)*

20 Side-Angle-Side (SAS) Congruence Postulate If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent. *(p. 240)*

21 Angle-Side-Angle (ASA) Congruence Postulate If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent. *(p. 249)*

22 Angle-Angle (AA) Similarity Postulate If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar. *(p. 381)*

23 Arc Addition Postulate The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. *(p. 660)*

24 Area of a Square Postulate The area of a square is the square of the length of its side, or $A = s^2$. *(p. 720)*

25 Area Congruence Postulate If two polygons are congruent, then they have the same area. *(p. 720)*

26 Area Addition Postulate The area of a region is the sum of the areas of its nonoverlapping parts. *(p. 720)*

27 Volume of a Cube The volume of a cube is the cube of the length of its side, or $V = s^3$. *(p. 819)*

28 Volume Congruence Postulate If two polyhedra are congruent, then they have the same volume. *(p. 819)*

29 Volume Addition Postulate The volume of a solid is the sum of the volumes of all its nonoverlapping parts. *(p. 819)*