9.1 – Translate Figures and Use

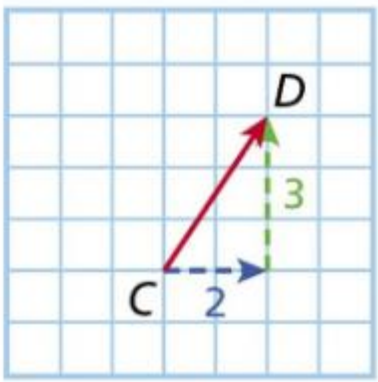
Image – An image is a new figure produced from the transformation of a figure.

Preimage – A preimage is the original figure in the transformation of a figure.

Isometry – An isometry is a transformation that preserves length and angle measure.

Vector – A vector is a quantity that has both direction and magnitude (size).

A vector can also be named using *component form*. The **component form** <x, y> of a vector lists the horizontal and **vertical** change from the initial point to the terminal point. The component form of $\overline{CD}$ is <2, **3**>.



THEOREM 9.1: TRANSLATION THEOREM

A translation is an isometry.

Your Notes

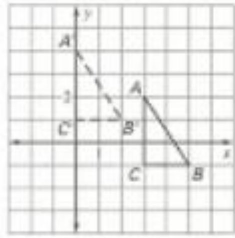
You can use *prime notation* to name an image. For example, if the preimage is △ABC, then its image is △A'B'C', read as "*triangle A prime, B prime, C prime.*"

Example 1 *Translate a figure in the coordinate plane*

Graph quadrilateral *ABCD* with vertices $A(-2, 6)$, $B(2, 4)$, $C(2, 1)$, and $D(-2, 3)$. Find the image of each vertex after the translation $(x, y) \rightarrow (x + 3, y - 3)$. Then graph the image using prime notation.

Example 2 *Write a translation rule and verify congruence*

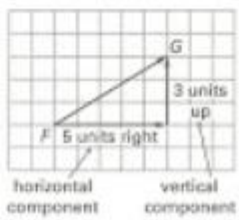
Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$. Then verify that the transformation is an isometry.



VECTORS

The diagram shows a vector named $\overrightarrow{FG}$, read as "vector *FG*."

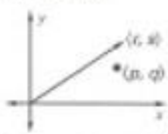
The initial point, or starting point, of the vector is ____.



The terminal point, or ending point, of the vector is ____.

The component form of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{FG}$ is ______.

Use brackets to write the component form of the vector $\langle r, s\rangle$. Use parentheses to write the coordinates of the point (p, q).



Your Notes

Example 3 *Identify vector components*

Name the vector and write its component form.

a.

b.

Example 4 *Use a vector to translate a figure*

The vertices of $\triangle ABC$ are $A(0, 4)$, $B(2, 3)$, and $C(1, 0)$. Translate $\triangle ABC$ using the vector $\langle -4, 1\rangle$.

9.2 – Use Properties of Matrices

- *Definition of Matrix*: A rectangular arrangement of numbers in rows and columns.
- Ex): Matrix A below has two rows and three columns.

A=

$$\begin{bmatrix} 6 & 2 & -1 \\ -2 & 0 & 5 \end{bmatrix}$$

2 rows

3 columns

Note: * The **DIMENSIONS** of matrix **A** are 2 X 3 (read "2 by 3")

* The numbers in a matrix are its ***ENTRIES.***

Ex.) The entry in the second row and third column is **5**.

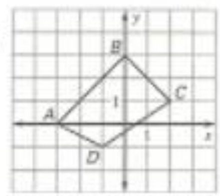
What is a matrix?

Example 1 *Represent figures using matrices*

Write a matrix to represent the point or polygon.

a. Point *A*

b. Quadrilateral *ABCD*



Your Notes

Example 2 *Add and subtract matrices*

a. $\begin{bmatrix} 4 & -2 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 5 & -6 \end{bmatrix}$

b. $\begin{bmatrix} 7 & 4 & 5 \\ 1 & -2 & 8 \end{bmatrix} - \begin{bmatrix} 3 & -6 & 5 \\ 0 & 7 & 1 \end{bmatrix}$

In order to add two matrices, they must have the same dimensions, so the translation matrix here must have three columns like the polygon matrix.

Example 3 *Represent a translation using matrices*

The matrix $\begin{bmatrix} 2 & 3 & 4 \\ -3 & 2 & 0 \end{bmatrix}$ represents $\triangle ABC$. Find the image matrix that represents the translation of $\triangle ABC$ 4 units left and 1 unit down. Then graph $\triangle ABC$ and its image.

Example 4 *Multiply matrices*

Multiply $\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix}$.

9.3 – Perform Reflections

Line of reflection – In a reflection, the mirror line is called the line of reflection.

Example 1 ***Graph reflections in horizontal and vertical lines***

The vertices of $\triangle ABC$ are $A(1, 2)$, $B(3, 0)$, and $C(5, 3)$. Graph the reflection of $\triangle ABC$ described.

a. In the line n: $x = 2$ **b.** In the line m: $y = 3$

Example 2 ***Graph a reflection in $y = x$***

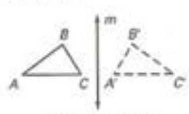
The endpoints of $\overline{CD}$ are $C(-2, 2)$ and $D(1, 2)$. Reflect the segment in the line $y = x$. Graph the segment and its image.

COORDINATE RULES FOR REFLECTIONS

- If (a, b) is reflected in the x-axis, its image is the point (____, ____).
- If (a, b) is reflected in the y-axis, its image is the point (____, ____).
- If (a, b) is reflected in the line $y = x$, its image is the point (____, ____).
- If (a, b) is reflected in the line $y = -x$, its image is the point (____, ____).

THEOREM 9.2: REFLECTION THEOREM

A reflection is an isometry.

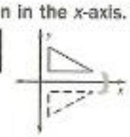


$\triangle ABC \cong \triangle A'B'C'$

REFLECTION MATRICES

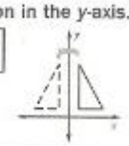
Reflection in the x-axis.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



Reflection in the y-axis.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



Your Notes

Example 5 ***Use matrix multiplication to reflect a polygon***

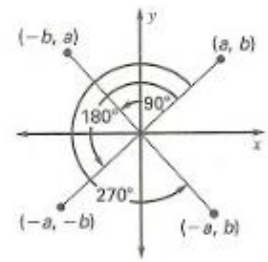
The vertices of $\triangle DEF$ are $D(1, 2)$, $E(2, 3)$, and $F(4, 1)$. Find the reflection of $\triangle DEF$ in the y-axis using matrix multiplication. Graph $\triangle DEF$ and its image.

9.4 – Perform Rotations

COORDINATE RULES FOR ROTATIONS ABOUT THE ORIGIN

When a point (a, b) is rotated counterclockwise about the origin, the following are true:

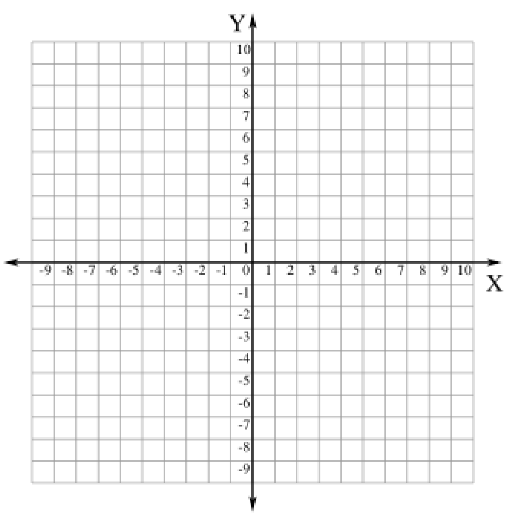
1. For a rotation of 90°, $(a, b) \rightarrow$ (____, ____).
2. For a rotation of 180°, $(a, b) \rightarrow$ (____, ____).
3. For a rotation of 270°, $(a, b) \rightarrow$ (____, ____).

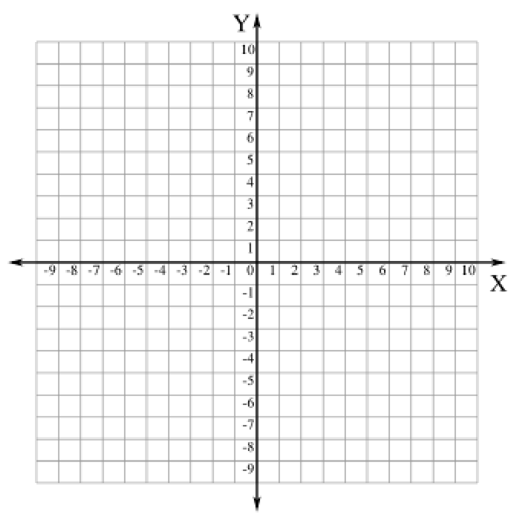


Example 2 *Rotate a figure using the coordinate rules*

Graph quadrilateral *KLMN* with vertices *K*(3, 2), *L*(4, 2), *M*(4, −3), and *N*(2, −1). Then rotate the quadrilateral 270° about the origin.

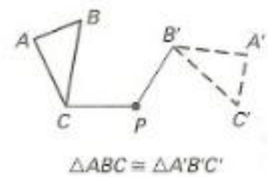
2. Graph *KLMN* in Example 2. Then rotate the quadrilateral 90° about the origin.





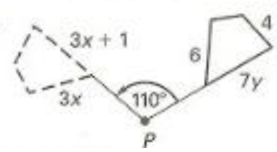
THEOREM 9.3: ROTATION THEOREM

A rotation is an isometry.

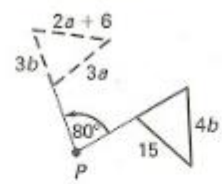


Example 4 *Find side lengths in a rotation*

The quadrilateral is rotated about P. Find the value of y.



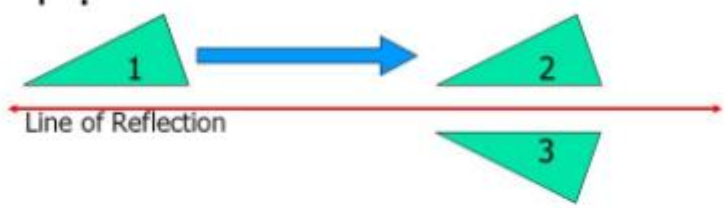
4. The triangle is rotated about P. Find the value of b.



9.5 – Apply Compositions of Transformations

Glide Reflection

1. A translation maps P onto P′.
2. A reflection in a line k parallel to the direction of the translation maps P′ to P″.



Example 1 *Find the image of a glide reflection*

The vertices of △ABC are A(2, 1), B(5, 3), and C(6, 2). Find the image of △ABC after the glide reflection.

Translation: $(x, y) \rightarrow (x - 8, y)$

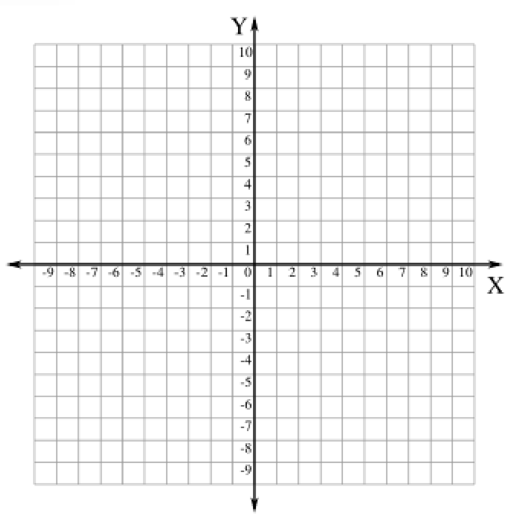
Reflection: in the x-axis

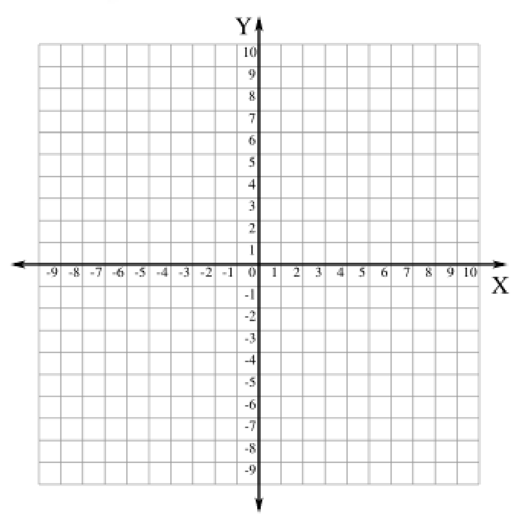
Example 2 *Find the image of a composition*

The endpoints of $\overline{CD}$ are C(−2, 6) and D(−1, 3). Graph the image of $\overline{CD}$ after the composition.

Reflection: in the y-axis

Rotation: 90° about the origin





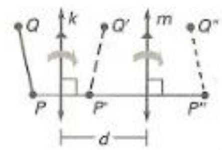
THEOREM 9.4: COMPOSITION THEOREM

The composition of two (or more) isometries is an isometry.

THEOREM 9.5: REFLECTIONS IN PARALLEL LINES THEOREM

If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a ____________.

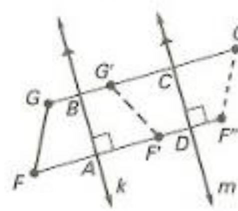
If P'' is the image of P, then:



1. $\overline{PP''}$ is perpendicular to k and m, and
2. $PP'' = 2d$, where d is the distance between k and m.

Example 3 *Use Theorem 9.5*

In the diagram, a reflection in line k maps $\overline{GF}$ to $\overline{G'F'}$. A reflection in line m maps $\overline{G'F'}$ to $\overline{G''F''}$. Also, $FA = 6$ and $DF'' = 3$.



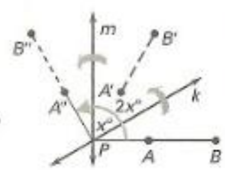
a. Name any segments congruent to each segment: $\overline{GF}$, $\overline{FA}$, and $\overline{GB}$.

b. Does $AD = BC$? *Explain.*

c. What is the length of $\overline{GG''}$?

THEOREM 9.6: REFLECTIONS IN INTERSECTING LINES THEOREM

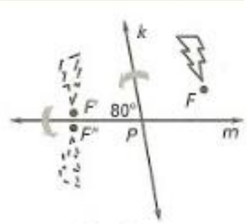
If lines k and m intersect at point P, then a reflection in k followed by a reflection in m is the same as a _________ about ____



The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by k and m.

Example 4 *Use Theorem 9.6*

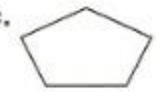
In the diagram, the figure is reflected in line k. The image is then reflected in line m. Describe a single transformation that maps F to F''.



9.6 – Identify Symmetry

Example 1 ***Identify lines of symmetry***

How many lines of symmetry does the figure have?

a.  **b.**  **c.** 

Your Notes

Example 2 ***Identify rotational symmetry***

Does the figure have rotational symmetry? If so, *describe* any rotations that map the figure onto itself.

a. Square 

b. Regular hexagon 

c. Kite 

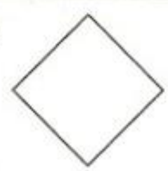
Example 3 ***Identify symmetry***

Identify the line symmetry and rotational symmetry of the figure at the right.

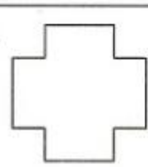
Your Notes

Checkpoint How many lines of symmetry does the figure have?

1. 

2. 

In Exercises 3 and 4, does the figure have rotational symmetry? If so, *describe* any rotations that map the figure onto itself.

3. 

4. 

5. Describe the lines of symmetry and rotational symmetry of the figure at the right.

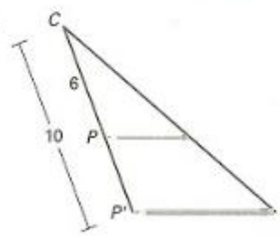
9.7 – Identify and Perform Dilations

Scalar multiplication – Scalar multiplication is the process of multiplying each element of a matrix by a real number or scalar.

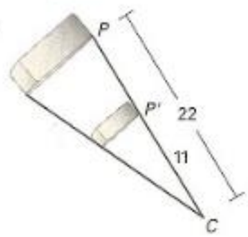
Example 1 *Identify dilations*

Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.

a.



b.



Checkpoint Simplify the product.

3. $4\begin{bmatrix} -6 & 3 & 2 \\ 5 & -1 & 4 \end{bmatrix}$	**4.** $-3\begin{bmatrix} 5 & -1 & -2 \\ -2 & 0 & 4 \end{bmatrix}$

Example 4 *Use scalar multiplication in a dilation*

The vertices of quadrilateral *ABCD* are *A*(−3, 0), *B*(0, 6), *C*(3, 6), and *D*(3, 3). Use scalar multiplication to find the image of *ABCD* after a dilation with its center at the origin and a scale factor of $\frac{1}{3}$. Graph *ABCD* and its image.

Example 5 *Find the image of a composition*

The vertices of △*KLM* are *K*(−3, 0), *L*(−2, 1), and *M*(−1, −1). Find the image of △*KLM* after the given composition.

Translation: $(x, y) \to (x + 4, y + 2)$
Dilation: centered at the origin with a scale factor of 2

Homework

6. A segment has the endpoints *C*(−2, 2) and *D*(2, 2). Find the image of $\overline{CD}$ after a 90° rotation about the origin followed by a dilation with its center at the origin and a scale factor of 2.