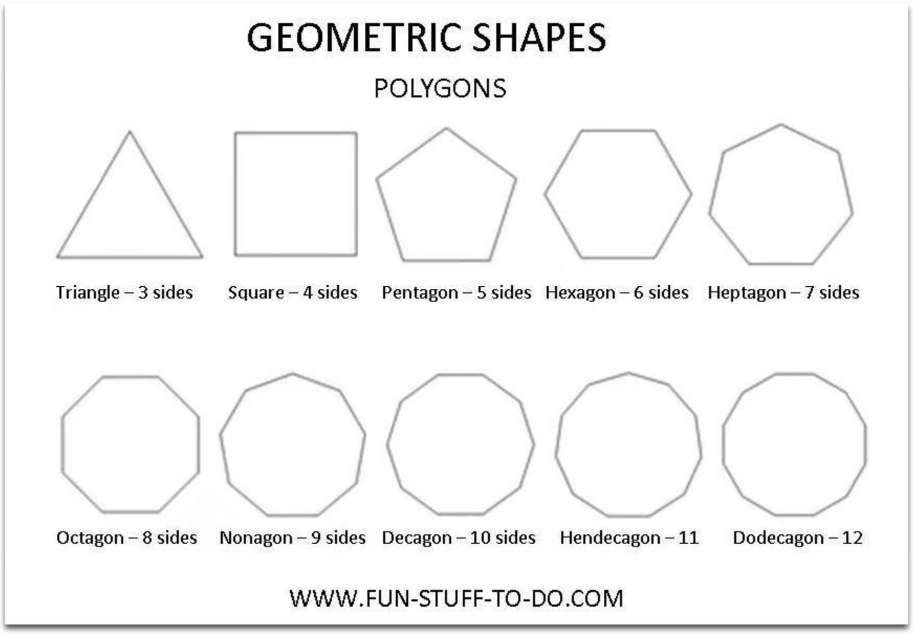
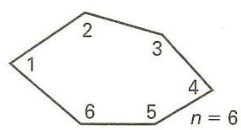
8.1 – Find Angle Measures in Polygons



THEOREM 8.1: POLYGON INTERIOR ANGLES THEOREM

The sum of the measures of the interior angles of a convex *n*-gon is (n – ___) • ______.



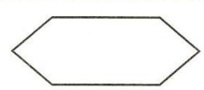
$m\angle 1 + m\angle 2 + \cdots + m\angle n$ = (n – ___) • ______

COROLLARY TO THEOREM 8.1: INTERIOR ANGLES OF A QUADRILATERAL

The sum of the measures of the interior angles of a quadrilateral is ______.

Example 1 ***Find the sum of angle measures in a polygon***

Find the sum of the measures of the interior angles of a convex hexagon.



Solution

A hexagon has ___ sides. Use the Polygon Interior Angles Theorem.

(n – ___) • ______ = (___ – ___) • ______ **Substitute __ for *n*.**

= ___ • ______ **Subtract.**

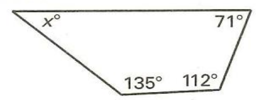
= ______ **Multiply.**

Example 2 ***Find the number of sides of a polygon***

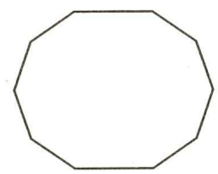
The sum of the measures of the interior angles of a convex polygon is 1260°. Classify the polygon by the number of sides.

Example 3 ***Find an unknown interior angle measure***

Find the value of *x* in the diagram shown.

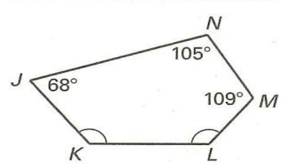


1. Find the sum of the measures of the interior angles of the convex decagon.



2. The sum of the measures of the interior angles of a convex polygon is 1620°. Classify the polygon by the number of sides.

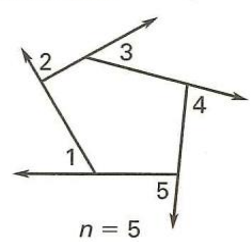
3. Use the diagram at the right. Find $m\angle K$ and $m\angle L$.



THEOREM 8.2: POLYGON EXTERIOR ANGLES THEOREM

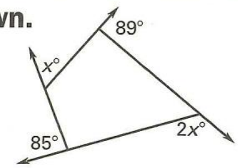
The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is ______.

$m\angle 1 + m\angle 2 + \cdots + m\angle n =$ ______



Example 4 ***Find unknown exterior angle measures***

Find the value of x in the diagram shown.



Example 5 ***Find angle measures in regular polygons***

Lamps The base of a lamp is in the shape of a regular 15-gon. Find (a) the measure of each interior angle and (b) the measure of each exterior angle.

4. A convex pentagon has exterior angles with measures 66°, 77°, 82°, and 62°. What is the measure of an exterior angle at the fifth vertex?

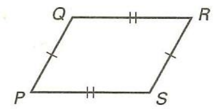
5. Find the measure of (a) each interior angle and (b) each exterior angle of a regular nonagon.

8.2 – Use Properties of Parallelograms

Parallelogram – a parallelogram is a quadrilateral with both pairs of opposite sides parallel.

THEOREM 8.3

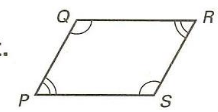
If a quadrilateral is a parallelogram, then its opposite sides are congruent.



If *PQRS* is a parallelogram, then _____ $\cong \overline{RS}$ and $\overline{QR} \cong$ _____.

THEOREM 8.4

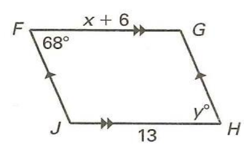
If a quadrilateral is a parallelogram, then its opposite angles are congruent.



If *PQRS* is a parallelogram, then $\angle P \cong$ _____ and _____ $\cong \angle S$.

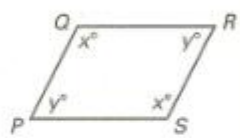
Example 1 ***Use properties of parallelograms***

Find the values of *x* and *y*.



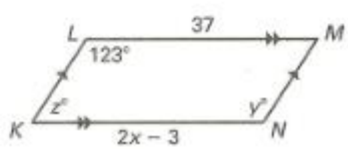
THEOREM 8.5

If a quadrilateral is a parallelogram, then its consecutive angles are _______________.



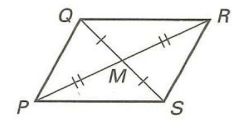
If *PQRS* is a parallelogram, then $x° + y° =$ _____.

Checkpoint **Find the indicated measure in ▱*KLMN* shown at the right.**



THEOREM 8.6

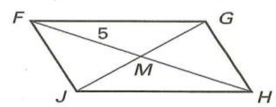
If a quadrilateral is a parallelogram, then its diagonals ________ each other.



$\overline{QM} \cong$ _____ and

$\overline{PM} \cong$ _____

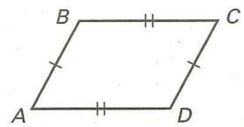
5. Given that ▱*FGHJ* is a parallelogram, find *MH* and *FH*.



8.3 – Show that a Quadrilateral is a Parallelogram

THEOREM 8.7

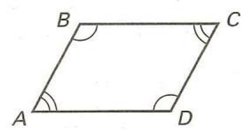
If both pairs of opposite ________ of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\overline{AB} \cong$ ______ and $\overline{BC} \cong$ ______, then *ABCD* is a parallelogram.

THEOREM 8.8

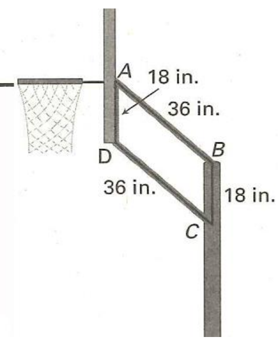
If both pairs of opposite ___ _____ of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\angle A \cong$ ______ and $\angle B \cong$ ______, then *ABCD* is a parallelogram.

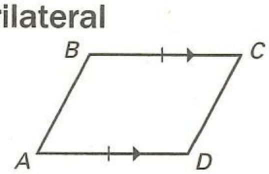
Example 1 ***Solve a real-world problem***

Basketball In the diagram at the right, $\overline{AB}$ and $\overline{DC}$ represent adjustable supports of a basketball hoop. *Explain* why $\overline{AD}$ is always parallel to $\overline{BC}$.



THEOREM 8.9

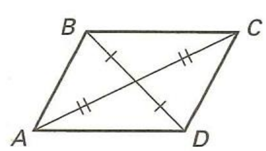
If one pair of opposite sides of a quadrilateral are ______________ and ___________, then the quadrilateral is a parallelogram.



If $\overline{BC}$ ___ $\overline{AD}$ and $\overline{BC}$ ____ $\overline{AD}$, then *ABCD* is a parallelogram.

THEOREM 8.10

If the diagonals of a quadrilateral _________ each other, then the quadrilateral is a parallelogram.



If $\overline{BD}$ and $\overline{AC}$ _________ each other, then *ABCD* is a parallelogram.

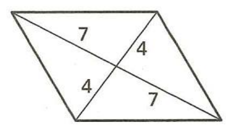
Example 2 *Identify a parallelogram*

Lights The headlights of a car have the shape shown at the right. *Explain* how you know that $\angle B \cong \angle D$.

Checkpoint Complete the following exercises.

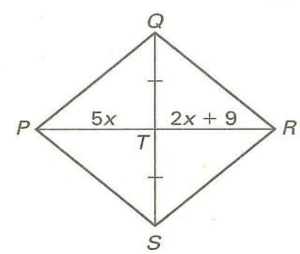
1. In quadrilateral $GHJK$, $m\angle G = 55°$, $m\angle H = 125°$, and $m\angle J = 55°$. Find $m\angle K$. What theorem can you use to show that $GHJK$ is a parallelogram?

2. What theorem can you use to show that the quadrilateral is a parallelogram?



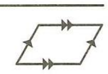
Example 3 *Use algebra with parallelograms*

For what value of x is quadrilateral $PQRS$ a parallelogram?

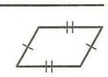


CONCEPT SUMMARY: WAYS TO PROVE A QUADRILATERAL IS A PARALLELOGRAM

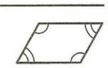
1. Show both pairs of opposite sides are parallel. **(Definition)**



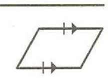
2. Show both pairs of opposite sides are congruent. **(Theorem 8.7)**



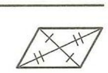
3. Show both pairs of opposite angles are congruent. **(Theorem 8.8)**



4. Show one pair of opposite sides are congruent and parallel. **(Theorem 8.9)**



5. Show the diagonals bisect each other. **(Theorem 8.10)**



8.4 – Show that a Quadrilateral is a Parallelogram

Rhombus – A rhombus is a parallelogram with four congruent sides.

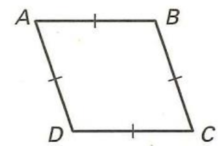
Rectangle – A rectangle is a parallelogram with four right angles.

Square – A square is a parallelogram with four congruent sides and four right angles.

RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent _______.

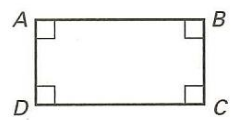
ABCD is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.



RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four ______________.

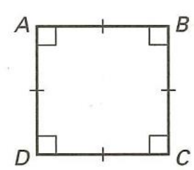
ABCD is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



SQUARE COROLLARY

A quadrilateral is a square if and only if it is a ___________ and a ___________.

ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



Example 1 ***Use properties of special quadrilaterals***

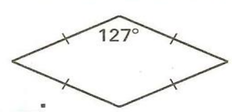
For any rhombus *RSTV*, decide whether the statement is always or sometimes true. Draw a sketch and explain your reasoning.

a. $\angle S \cong \angle V$

b. $\angle T \cong \angle V$

Example 2 ***Classify special quadrilaterals***

Classify the special quadrilateral.
Explain **your reasoning.**



The quadrilateral has four congruent ________. One of the angles is not a ___ _, so the rhombus is not also a __ ___. By the Rhombus Corollary, the quadrilateral is a _ ______.

1. For any square *CDEF*, is it *always* or *sometimes* true that $\overline{CD} \cong \overline{DE}$? *Explain* your reasoning.

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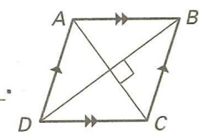
2. A quadrilateral has four congruent sides and four congruent angles. Classify the quadrilateral.

THEOREM 8.11

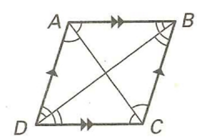
A parallelogram is a rhombus if and only if its diagonals are ____________.



$\square ABCD$ is a rhombus if and only if ____ $\perp$ ____.

THEOREM 8.12

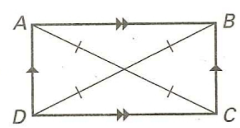
A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.



$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle$____ and $\angle$____ and $\overline{BD}$ bisects $\angle$____ and $\angle$____.

THEOREM 8.13

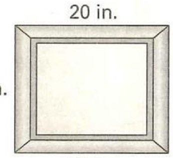
A parallelogram is a rectangle if and only if its diagonals are ________.



$\square ABCD$ is a rectangle if and only if ____ $\cong$ ____.

Example 4 ***Solve a real-world problem***

Framing You are building a frame for a painting. The measurements of the frame are shown at the right.



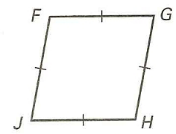
a. The frame must be a rectangle. Given the measurements in the diagram, can you assume that it is? *Explain*.

b. You measure the diagonals of the frame. The diagonals are about 25.6 inches. What can you conclude about the shape of the frame?

Example 3 *List properties of special parallelograms*

Sketch rhombus *FGHJ*. List everything you know about it.

Solution



By definition, you need to draw a figure with the following properties:

- The figure is a ________ ______.
- The figure has four congruent _______.

Because *FGHJ* is a parallelogram, it has these properties:

- Opposite sides are _________ and ____________.
- Opposite angles are ____________. Consecutive angles are _____________ __.
- Diagonals _______ each other.

By Theorem 8.11, the diagonals of *FGHJ* are ____ __________. By Theorem 8.12, each diagonal bisects a pair of _ .

3. Sketch rectangle *WXYZ*. List everything that you know about it.

4. Suppose the diagonals of the frame in Example 4 are not congruent.

Could the frame still be a rectangle? *Explain*.

8.5 – Use Properties of Trapezoids and Kites

Trapezoid – A trapezoid is a quadrilateral with exactly one pair of parallel sides.

Bases of a trapezoid – The parallel sides of a trapezoid are the bases.

Base angles of a trapezoid – A trapezoid has 2 pairs of base angles. Each pair shares a base as a side.

Legs of a trapezoid – The nonparallel sides of a trapezoid are the legs.

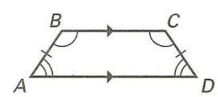
Isosceles trapezoid – An isosceles trapezoid is a trapezoid in which the legs are congruent.

Midsegment of a trapezoid – The midsegment of a trapezoid is the segment that connects the midpoints of its legs.

Kite – a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

THEOREM 8.14

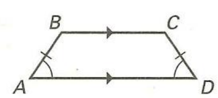
If a trapezoid is isosceles, then each pair of base angles is ____________.



If trapezoid *ABCD* is isosceles, then $\angle A \cong \angle$ ___ and $\angle$ ___ $\cong \angle C$.

THEOREM 8.15

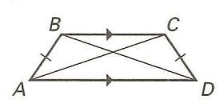
If a trapezoid has a pair of congruent ______________, then it is an isosceles trapezoid.



If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid *ABCD* is isosceles.

THEOREM 8.16

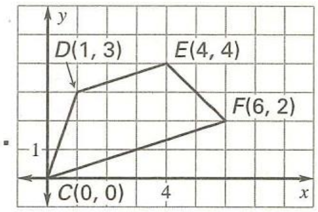
A trapezoid is isosceles if and only if its diagonals are ____________.



Trapezoid *ABCD* is isosceles if and only if _____ $\cong$ _____.

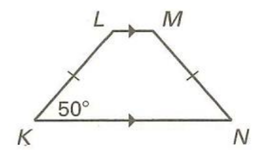
Example 1 ***Use a coordinate plane***

Show that *CDEF* is a trapezoid.



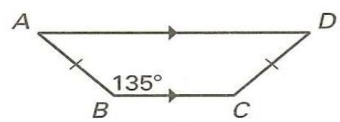
Example 2 ***Use properties of isosceles trapezoids***

Kitchen A shelf fitting into a cupboard in the corner of a kitchen is an isosceles trapezoid. Find $m\angle N$, $m\angle L$, and $m\angle M$.



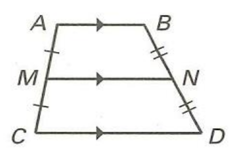
1. In Example 1, suppose the coordinates of point *E* are (7, 5). What type of quadrilateral is *CDEF*? *Explain.*

2. Find $m\angle C$, $m\angle A$, and $m\angle D$ in the trapezoid shown.



THEOREM 8.17: MIDSEGMENT THEOREM FOR TRAPEZOIDS

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

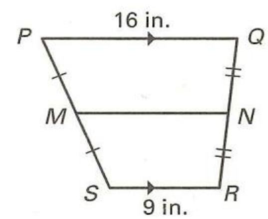


If $\overline{MN}$ is the midsegment of trapezoid *ABCD*, then

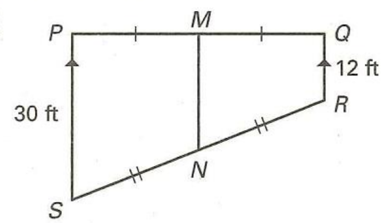
$\overline{MN} \parallel$ _____, $\overline{MN} \parallel$ _____, and $MN = \frac{\quad}{\quad}$(_____ + _____).

Example 3 ***Use the midsegment of a trapezoid***

In the diagram, $\overline{MN}$ is the midsegment of trapezoid *PQRS*. Find *MN*.



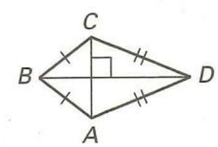
3. Find *MN* in the trapezoid at the right.



THEOREM 8.18

If a quadrilateral is a kite, then its diagonals are ________________.

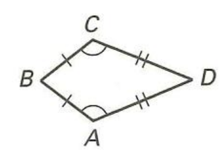
If quadrilateral *ABCD* is a kite, then _____ ⊥ _____.



THEOREM 8.19

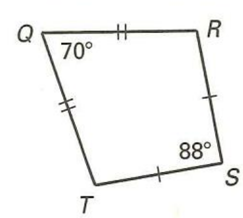
If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral *ABCD* is a kite and $\overline{BC} \cong \overline{BA}$, then ∠*A* ____ ∠*C* and ∠*B* ____ ∠*D*.



Example 4 ***Apply Theorem 8.19***

Find *m*∠*T* in the kite shown at the right.



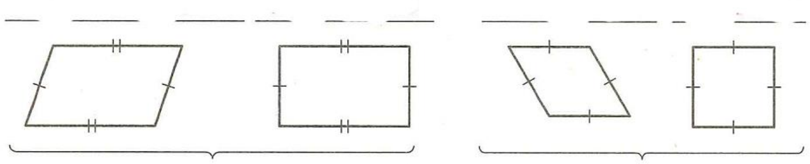
8.6 – Identify Special Quadrilaterals

Example 1 ***Identify quadrilaterals***

Quadrilateral *ABCD* has both pairs of opposite sides congruent. What types of quadrilaterals meet this condition?

Solution

There are many possibilities.

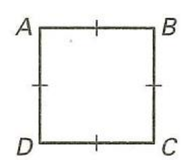


Opposite sides are congruent. All sides are congruent.

1. Quadrilateral *JKLM* has both pairs of opposite angles congruent. What types of quadrilaterals meet this condition?

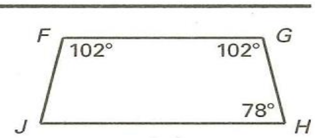
Example 2 ***Identify a quadrilateral***

What is the most specific name for quadrilateral *ABCD*?



Example 3 ***Identify a quadrilateral***

Is enough information given in the diagram to show that quadrilateral *FGHJ* is an isosceles trapezoid? *Explain*.



2. What is the most specific name for quadrilateral *QRST*? *Explain* your reasoning.