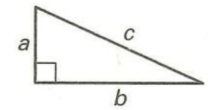
7.1 – Apply the Pythagorean Theorem

THEOREM 7.1: PYTHAGOREAN THEOREM

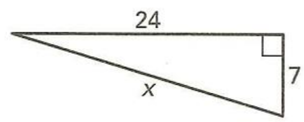
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$

Example 1 ***Find the length of a hypotenuse***

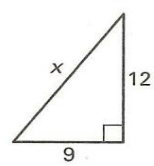
Find the length of the hypotenuse of the right triangle.



Solution

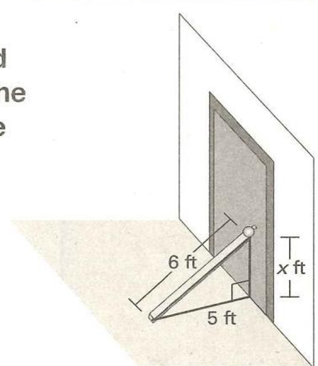
$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ **Pythagorean Theorem**

1. Find the length of the hypotenuse of the right triangle.



Example 2 ***Find the length of a leg***

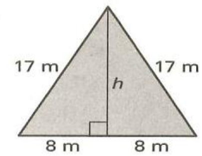
Door A 6 foot board rests under a doorknob and the base of the board is 5 feet away from the bottom of the door. Approximately how high above the ground is the doorknob?



2. A 5 foot board rests under a doorknob and the base of the board is 3.5 feet away from the bottom of the door. Approximately how high above the ground is the doorknob?

Example 3 *Find the area of an isosceles triangle*

Find the area of the isosceles triangle with side lengths 16 meters, 17 meters, and 17 meters.



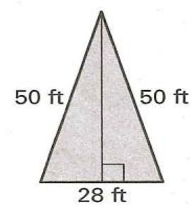
You may find it helpful to memorize the basic Pythagorean triples, shown in **bold**, for standardized tests.

COMMON PYTHAGOREAN TRIPLES AND SOME OF THEIR MULTIPLES

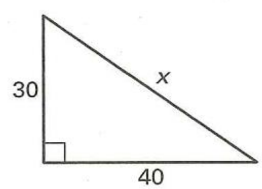
3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x$, $4x$, $5x$	$5x$, $12x$, $13x$	$8x$, $15x$, $17x$	$7x$, $24x$, $25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

3. Find the area of the triangle.

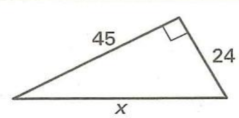


4. Use a Pythagorean triple to find the unknown side length of the right triangle.



Example 4 *Find length of a hypotenuse using two methods*

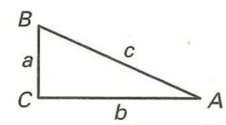
Find the length of the hypotenuse of the right triangle.



7.2 – Use the Converse of the Pythagorean Theorem

THEOREM 7.2: CONVERSE OF THE PYTHAGOREAN THEOREM

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a ___ ___ triangle.

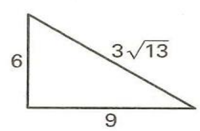


If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a ______ triangle.

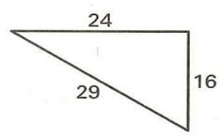
Example 1 ***Verify right triangles***

Tell whether the given triangle is a right triangle.

a.

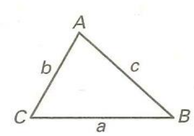


b.



THEOREM 7.3

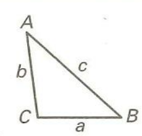
If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an _______ triangle.



If $c^2 < a^2 + b^2$, then the triangle *ABC* is _______.

THEOREM 7.4

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an ________ triangle.



If $c^2 > a^2 + b^2$, then the triangle *ABC* is ________.

Example 2 *Classify triangles*

Can segments with lengths of 2.8 feet, 3.2 feet, and 4.2 feet form a triangle? If so, would the triangle be *acute*, *right*, or *obtuse*?

Solution

> The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Step 1 **Use** the Triangle Inequality Theorem to check that the segments can make a triangle.

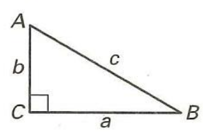
$2.8 + 3.2 =$ ___	$2.8 + 4.2 =$ ___	$3.2 + 4.2 =$ ___
___ > 4.2	___ > 3.2	___ > 2.8

Step 2 **Classify** the triangle by comparing the square of the length of the longest side with the sum of squares of the lengths of the shorter sides.

$c^2 \underline{\ ?\ } a^2 + b^2$ Compare c^2 with $a^2 + b^2$.

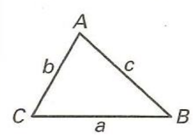
METHODS FOR CLASSIFYING A TRIANGLE BY ANGLES USING ITS SIDE LENGTHS

Theorem 7.2



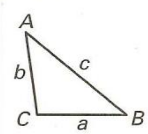
If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a ______ triangle.

Theorem 7.3



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an ______ triangle.

Theorem 7.4

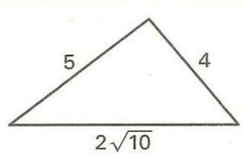


If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an ______ triangle.

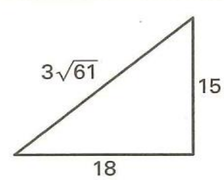
lotes

✔ *Checkpoint* In Exercises 1 and 2, tell whether the triangle is a right triangle.

1.



2.

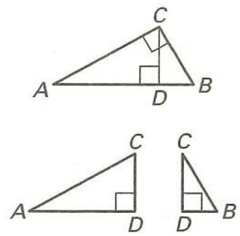


3. Can segments with lengths of 6.1 inches, 9.4 inches, and 11.3 inches form a triangle? If so, would the triangle be *acute*, *right*, or *obtuse*?

7.3 – Use Similar Right Triangles

THEOREM 7.5

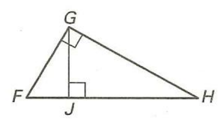
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are ________ to the original triangle and to each other.



$\triangle CBD$ ____ $\triangle ABC$, $\triangle ACD$ ____ $\triangle ABC$, and $\triangle CBD$ ____ $\triangle ACD$.

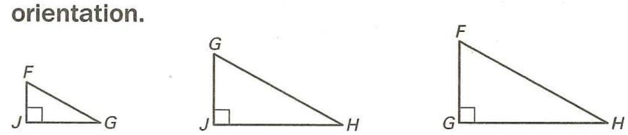
Example 1 ***Identify similar triangles***

Identify the similar triangles in the diagram.



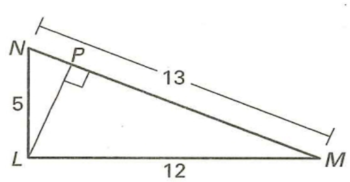
Solution

Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.



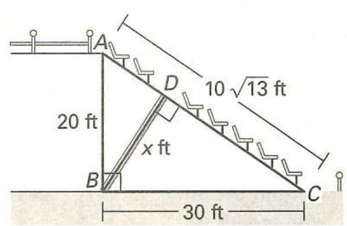
$\triangle$______ $\sim$ $\triangle$______ $\sim$ $\triangle$______

1. Identify the similar triangles in the diagram.

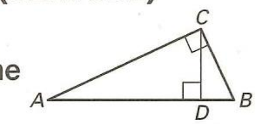


Example 2 ***Find the length of the altitude to the hypotenuse***

Stadium A cross section of a group of seats at a stadium shows a drainage pipe $\overline{BD}$ that leads from the seats to the inside of the stadium. What is the length of the pipe?



THEOREM 7.6: GEOMETRIC MEAN (ALTITUDE) THEOREM

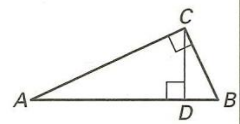


In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the ________________ of the lengths of the two segments.

$$\frac{BD}{\square} = \frac{\square}{AD}$$

THEOREM 7.7: GEOMETRIC MEAN (LEG) THEOREM



In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

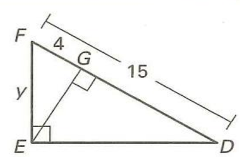
The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is ________ to the leg.

$$\frac{AB}{CB} = \frac{CB}{\square} \text{ and}$$

$$\frac{AB}{AC} = \frac{AC}{\square}$$

Example 3 *Use a geometric mean*

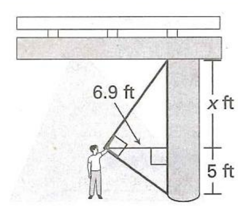
Find the value of y. Write your answer in simplest radical form.



Example 4 *Find a height using indirect measurement*

Overpass To find the clearance under an overpass, you need to find the height of a concrete support beam.

You use a cardboard square to line up the top and bottom of the beam. Your friend measures the vertical distance from the ground to your eye and the distance from you to the beam. Approximate the height of the beam.



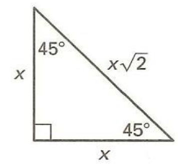
7.4 – Special Right Triangles

The extended ratio of the side lengths of a 45°-45°-90° triangle is $1:1:\sqrt{2}$.

THEOREM 7.8: 45°-45°-90° TRIANGLE THEOREM

In a 45°-45°-90° triangle, the hypotenuse is _____ times as long as each leg.

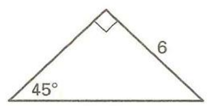
hypotenuse = leg • _____



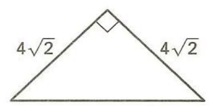
Example 1 *Find hypotenuse length in a 45°-45°-90° triangle*

Find the length of the hypotenuse.

a.

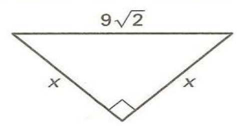


b.



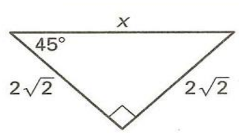
Example 2 *Find leg lengths in a 45°-45°-90° triangle*

Find the lengths of the legs in the triangle.

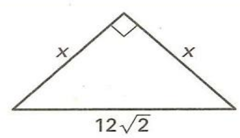


Checkpoint **Find the value of the variable.**

1.



2.



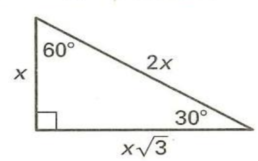
The extended ratio of the side lengths of a 30°-60°-90° triangle is $1:\sqrt{3}:2$.

THEOREM 7.9: 30°-60°-90° TRIANGLE THEOREM

In a 30°-60°-90° triangle, the hypotenuse is _______ as long as the shorter leg, and the longer leg is _____ times as long as the shorter leg.

hypotenuse = ____ • shorter leg

longer leg = shorter leg • _____



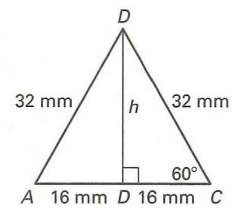
Your Notes

> Remember that in an equilateral triangle, the altitude to a side is also the median to that side. So, altitude $\overline{BD}$ ________ $\overline{AC}$.

Example 3 *Find the height of an equilateral triangle*

Music You make a guitar pick that resembles an equilateral triangle with side lengths of 32 millimeters. What is the approximate height of the pick?

Solution



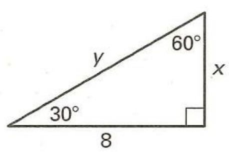
Draw the equilateral triangle described. Its altitude forms the longer leg of two ____-____-90° triangles. The length h of the altitude is approximately the height of the pick.

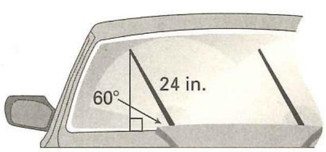
longer leg = shorter leg • ____

h = ____ • ____ ≈ _____ mm

Example 4 *Find lengths in a 30°-60°-90° triangle*

Find the values of x and y. Write your answer in simplest radical form.

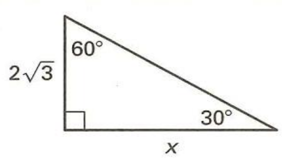




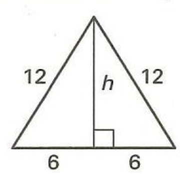
Example 5 *Find a height*

Windshield wipers A car is turned off while the windshield wipers are moving. The 24 inch wipers stop, making a 60° angle with the bottom of the windshield. How far from the bottom of the windshield are the ends of the wipers?

3.



4.



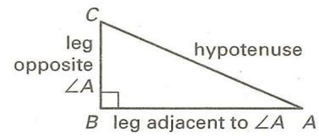
5. In Example 5, how far from the bottom of the windshield are the ends of the wipers if they make a 30° angle with the bottom of the windshield?

7.5 – Apply the Tangent Ratio

> Remember these abbreviations:
> tangent → tan
> opposite → opp.
> adjacent → adj.

TANGENT RATIO

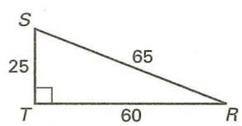
Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as tan A) is defined as follows:



$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{\square}{\square}$$

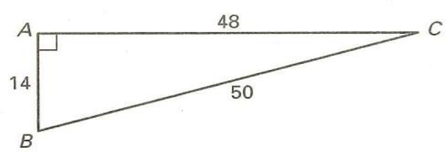
Example 1 ***Find tangent ratios***

Find tan S and tan R. Write each answer as a fraction and as a decimal rounded to four places, if necessary.



✔ ***Checkpoint*** **Find tan B and tan C. Write each answer as a fraction and as a decimal rounded to four places.**

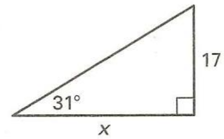
1.



Example 2 *Find a leg length*

Find the value of x.

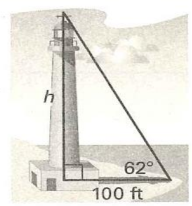
Use the tangent of an acute angle to find a leg length.



$\tan 31° = \frac{\_\_\_}{\_\_\_}$	**Write ratio for tangent of 31°.**
$\tan 31° = \frac{\_\_\_}{\_\_\_}$	**Substitute.**
___ • $\tan 31° =$ ___	**Multiply each side by ___.**
$x = \frac{\_\_\_}{\_\_\_}$	**Divide each side by ________.**
$x \approx \frac{\_\_\_}{\_\_\_\_\_\_}$	**Use a calculator to find ________.**
$x \approx$ ______	**Simplify.**

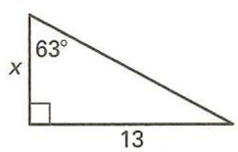
Example 3 *Estimate height using tangent*

Lighthouse Find the height h of the lighthouse to the nearest foot.

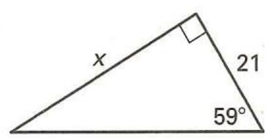


✓ *Checkpoint* In Exercises 2 and 3, find the value of x. Round to the nearest tenth.

2.



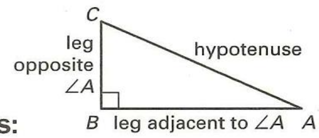
3.



7.6 – Apply the Sine and Cosine Ratio

SINE AND COSINE RATIOS

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written sin A and cos A) are defined as follows:



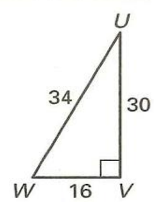
$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{\square}{\square}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{\square}{\square}$$

Remember these abbreviations:
sine → sin
cosine → cos
hypotenuse → hyp

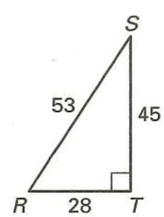
Example 1 ***Find sine ratios***

Find sin U and sin W. Write each answer as a fraction and as a decimal rounded to four places.



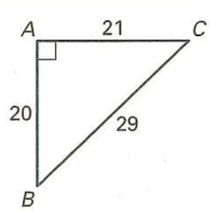
Example 2 ***Find cosine ratios***

Find cos S and cos R. Write each answer as a fraction and as a decimal rounded to four places.



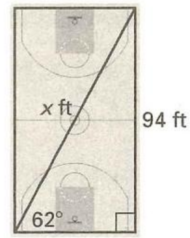
Checkpoint **Find sin B, sin C, cos B, and cos C. Write each answer as a fraction and as a decimal rounded to four places.**

1.



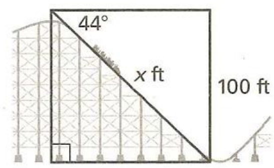
Example 3 *Use a trigonometric ratio to find a hypotenuse*

Basketball You walk from one corner of a basketball court to the opposite corner. Write and solve a proportion using a trigonometric ratio to approximate the distance of the walk.



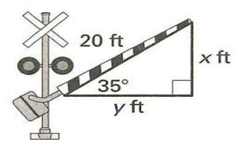
Example 4 *Find a hypotenuse using an angle of depression*

Roller Coaster You are at the top of a roller coaster 100 feet above the ground. The angle of depression is 44°. About how far do you ride down the hill?



Example 5 *Find leg lengths using an angle of elevation*

Railroad A railroad crossing arm that is 20 feet long is stuck with an angle of elevation of 35°. Find the lengths x and y.



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Example 6 *Use a special right triangle to find a sin and cos*

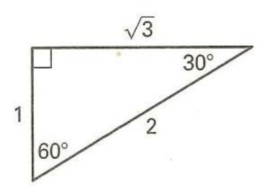
Use a special right triangle to find the sine and cosine of a 30° angle.

Solution

Use the 30°-60°-90° Triangle Theorem to draw a right triangle with side lengths of 1, $\sqrt{3}$, and ___. Then set up sine and cosine ratios for the 30° angle.

sin 30° = ___/___ = ___/___ = ________

cos 30° = ___/___ = ___/___ ≈ ________

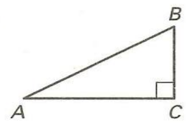


7.7 – Solve Right Triangles

The expression "$\tan^{-1} x$" is read as "the inverse tangent of x."

INVERSE TRIGONOMETRIC RATIOS

Let $\angle A$ be an acute angle.



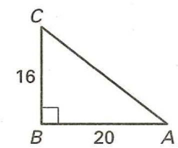
Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\qquad \tan^{-1} \frac{BC}{AC} = m\angle A$

Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\qquad \sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\qquad \cos^{-1} \frac{AC}{AB} = m\angle A$

Example 1 ***Use an inverse tangent to find an angle measure***

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Because $\tan A =$ ___ $=$ ___ $=$ ___,

$\tan^{-1}$ ___ $= m\angle A$. Using a calculator,

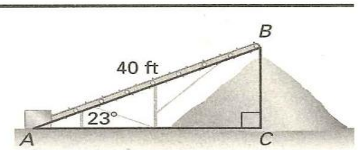
$\tan^{-1}$ ___ $\approx$ ___ ___.

So, the measure of $\angle A$ is approximately ___.

1. In Example 1, use a calculator and an inverse tangent to approximate $m\angle C$ to the nearest tenth of a degree.

Example 3 ***Solve a right triangle***

Solve the right triangle. Round decimal answers to the nearest tenth.



Example 4 ***Solve a real-world problem***

Model Train You are building a track for a model train. You want the track to incline from the first level to the second level, 4 inches higher, in 96 inches. Is the angle of elevation less than 3°?