6.1 – Ratios, Proportions, and the Geometric Mean

Ratio of a to b –

Proportion –

Means –

Geometric mean –

Example 1 *Simplify ratios*

Simplify the ratio. ***(See Table of Measures, p. 921)***

a. 76 cm : 8 cm

b. $\frac{4 \text{ ft}}{24 \text{ in.}}$

Checkpoint **In Exercises 1 and 2, simplify the ratio.**

1. 4 meters to 18 meters

2. 33 yd : 9 ft

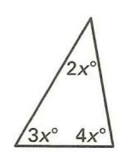
3. The perimeter of a rectangular table is 21 feet and the ratio of its length to its width is 5 : 2. Find the length and width of the table.

Example 3 *Use extended ratios*

The measures of the angles in $\triangle BCD$ are in the *extended ratio* of 2 : 3 : 4. Find the measures of the angles.

Solution

Begin by sketching the triangle. Then use the extended ratio of 2 : 3 : 4 to label the measures as ___$x°$, ___$x°$, and ___$x°$.



Checkpoint **Complete the following exercise.**

4. A triangle's angle measures are in the extended ratio of 1 : 4 : 5. Find the measures of the angles.

A PROPERTY OF PROPORTIONS

1. **Cross Products Property** In a proportion, the product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$, then ____ = ____.

$\frac{2}{3} = \frac{4}{6}$ $3 \cdot$ ___ = ___

$2 \cdot$ ___ = ___

Example 4 *Solve proportions*

Solve the proportion.

a. $\frac{3}{4} = \frac{x}{16}$

b. $\frac{3}{x+1} = \frac{2}{x}$

GEOMETRIC MEAN

The geometric mean of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$.

So, $x^2 =$ ____ and $x = \sqrt{\_\_\_\_}$.

Your Notes

Example 6 *Find a geometric mean*

Find the geometric mean of 16 and 48.

8. Find the geometric mean of 14 and 16.

6.2 – Use Proportions to Solve Geometry Problems

Scale drawing –

Scale –

ADDITIONAL PROPERTIES OF PROPORTIONS

2. **Reciprocal Property** If two ratios are equal, then their reciprocals are also equal.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} =$ ____.

3. If you interchange the means of a proportion, then you form another true proportion.

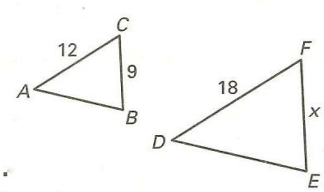
   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} =$ ____.

4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} =$ ____.

Example 1 *Use properties of proportions*

In the diagram, $\frac{AC}{DF} = \frac{BC}{EF}$. Write four true proportions.



Because $\frac{AC}{DF} = \frac{BC}{EF}$, then $\frac{12}{18} =$ ____.

Reciprocal Property: The reciprocals are equal, so $\frac{18}{12} =$ ____.

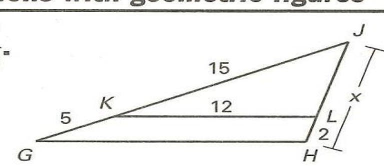
Property 3: You can interchange the means, so $\frac{12}{9} =$ ____.

Property 4: You can add the denominators to the numerators, so ____ = ____.

Example 2 *Use proportions with geometric figures*

In the diagram, $\frac{JL}{LH} = \frac{JK}{KG}$.

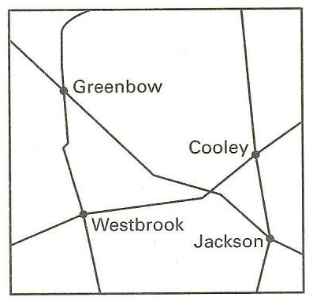
Find JH and JL.



1. In Example 1, find the value of x.	**2.** In Example 2, $\frac{KL}{GH} = \frac{JK}{JG}$. Find GH.

Example 4 ***Use a scale drawing***

Maps The scale of the map at the right is 1 inch : 8 miles. Find the actual distance from Westbrook to Cooley.



Solution

Use a ruler. The distance from Westbrook to Cooley on the map is about ___ ________. Let x be the actual distance in miles.

$$\frac{\underline{\qquad}}{\underline{\qquad}} = \frac{1\text{ in.}}{8\text{ mi}} \quad \begin{matrix} \leftarrow \text{distance on map} \\ \leftarrow \text{actual distance} \end{matrix}$$

$x =$ ___ ___ **Cross Products Property**

$x =$ _ _ **Simplify.**

Example 5 ***Solve a multi-step problem***

Scale Model You buy a 3-D scale model of the Sunsphere in Knoxville, TN. The actual building is 266 feet tall. Your model is 20 inches tall, and the diameter of the dome on your scale model is about 5.6 inches.

a. What is the diameter of the actual dome?

b. How many times as tall as your model is the actual building?

4. Two landmarks are 130 miles from each other. The landmarks are 6.5 inches apart on a map. Find the scale of the map.

6.3 – Use Similar Polygons

Similar polygons –

Scale factor of two similar polygons – If two polygons are similar then the ration of the lengths of two corresponding sides is called a scale factor.

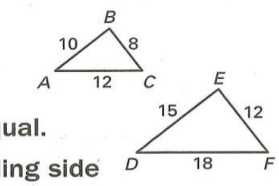
Example 1 ***Use similarity statements***

In the diagram, $\triangle ABC \sim \triangle DEF$.

a. List all pairs of congruent angles.

b. Check that the ratios of corresponding side lengths are equal.

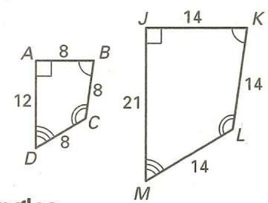
c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.



In a *statement of proportionality*, any pair of ratios forms a true proportion.

Example 2 ***Find the scale factor***

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *ABCD* to *JKLM*.



Solution

Step 1 **Identify pairs of congruent angles.**
From the diagram, you can see that $\angle B \cong \angle$___, $\angle C \cong \angle$___, and $\angle D \cong \angle$___. Angles ___ and ___ are right angles, so $\angle$___ $\cong \angle$___. So, the corresponding angles are ___________.

Step 2 **Show** that corresponding side lengths are proportional.

$\frac{AB}{JK} = \underline{\qquad} = \underline{\quad}$ $\frac{BC}{KL} = \underline{\qquad} = \underline{\quad}$

$\frac{CD}{LM} = \underline{\qquad} = \underline{\quad}$ $\frac{AD}{JM} = \underline{\qquad} = \underline{\quad}$

The ratios are equal, so the corresponding side lengths are ______________.

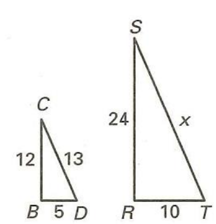
So *ABCD* $\sim$ ________. The scale factor of *ABCD* to *JKLM* is ___.

Example 3 ***Use similar polygons***

In the diagram, $\triangle BCD \sim \triangle RST$. Find the value of x.

Solution

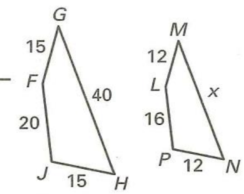
The triangles are similar, so the corresponding side lengths are ___________.



Checkpoint In the diagram, $FGHJ \sim LMNP$.

2. What is the scale factor of $LMNP$ to $FGHJ$?

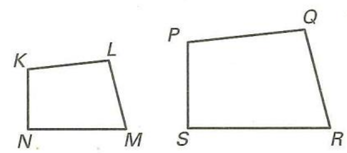
3. Find the value of x.



THEOREM 6.1: PERIMETERS OF SIMILAR POLYGONS

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

If $KLMN \sim PQRS$, then



$$\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \underline{\qquad} = \underline{\qquad} = \underline{\qquad} = \underline{\qquad}.$$

Example 4 *Find perimeters of similar figures*

Basketball A larger cement court is being poured for a basketball hoop in place of a smaller one. The court will be 20 feet wide and 25 feet long. The old court was similar in shape, but only 16 feet wide.

a. Find the scale factor of the new court to the old court.

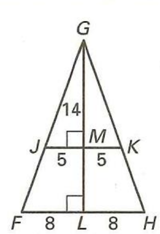
b. Find the perimeters of the new court and the old court.

CORRESPONDING LENGTHS IN SIMILAR POLYGONS

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the ____________ of the similar polygons.

Example 5 *Use a scale factor*

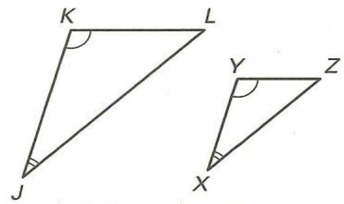
In the diagram, $\triangle FGH \sim \triangle JGK$. Find the length of the altitude $\overline{GL}$.



6.4 – Prove Triangles Similar by AA

POSTULATE 22: ANGLE-ANGLE (AA) SIMILARITY POSTULATE

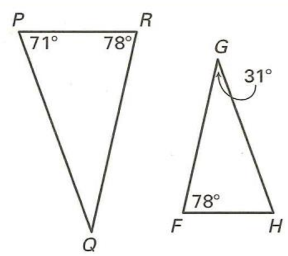
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



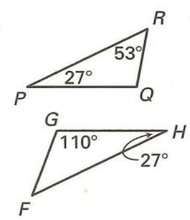
$\triangle JKL \sim \triangle XYZ$

Checkpoint Determine whether the triangles are similar. If they are, write a similarity statement.

1.



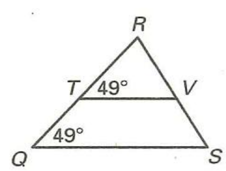
2.



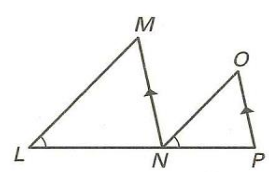
Example 2 ***Show that triangles are similar***

Show that the two triangles are similar.

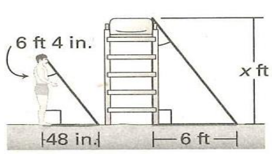
a. $\triangle RTV$ and $\triangle RQS$



b. $\triangle LMN$ and $\triangle NOP$



Example 3 ***Using similar triangles***



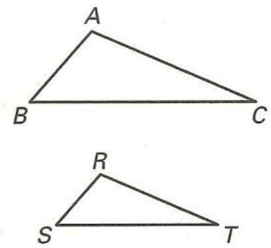
Height A lifeguard is standing beside the lifeguard chair on a beach. The lifeguard is 6 feet 4 inches tall and casts a shadow that is 48 inches long. The chair casts a shadow that is 6 feet long. How tall is the chair?

6.5 – Prove Triangles Similar by SSS and SAS

THEOREM 6.2: SIDE-SIDE-SIDE (SSS) SIMILARITY THEOREM

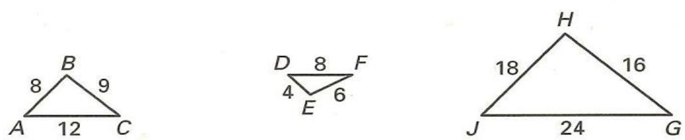
If the corresponding side lengths of two triangles are _______ ____, then the triangles are similar.

If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.



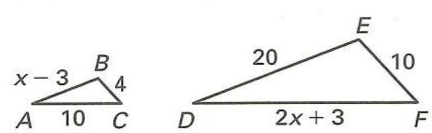
Example 1 *Use the SSS Similarity Theorem*

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



Example 2 *Use the SSS Similarity Theorem*

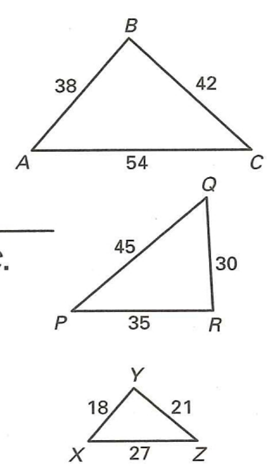
Find the value of x that makes $\triangle ABC \sim \triangle DEF$.



Checkpoint **Complete the following exercises.**

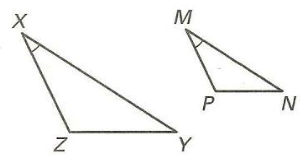
1. Which of the three triangles are similar?

2. Suppose AB is not given in $\triangle ABC$. What length for AB would make $\triangle ABC$ similar to $\triangle QRP$?



THEOREM 6.3: SIDE-ANGLE-SIDE (SAS) SIMILARITY THEOREM

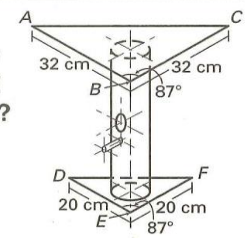
If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are ______ ______, then the triangles are similar.



If $\angle X \cong \angle M$, and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Example 3 *Use the SAS Similarity Theorem*

Birdfeeder You are drawing a design for a birdfeeder. Can you construct the top so it is similar to the bottom using the angle measure and lengths shown?



TRIANGLE SIMILARITY POSTULATE AND THEOREMS

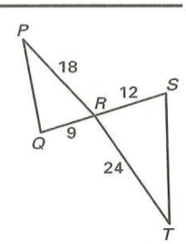
AA Similarity Postulate If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

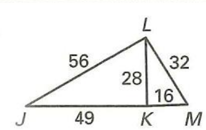
Example 4 *Choose a method*

Tell what method you would use to show that the triangles are similar.



Solution

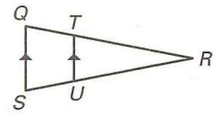
4. *Explain* how to show $\triangle JKL \sim \triangle LKM$.



6.6 – Use Proportionality Theorems

THEOREM 6.4: TRIANGLE PROPORTIONALITY THEOREM

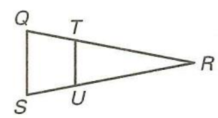
If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides ___ ______.



If $\overline{TU} \parallel \overline{QS}$, then $\_\_\_ = \_\_\_$.

THEOREM 6.5: CONVERSE OF THE TRIANGLE PROPORTIONALITY THEOREM

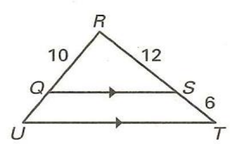
If a line divides two sides of a triangle proportionally, then it is parallel to the ____________.



If $\frac{RT}{TQ} = \frac{RU}{US}$, then ____ $\parallel$ ____.

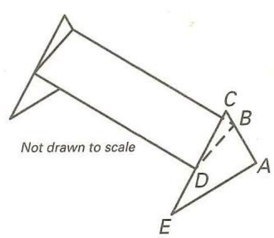
Example 1 *Find the length of a segment*

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RQ = 10$, $RS = 12$, and $ST = 6$. What is the length of $\overline{QU}$?

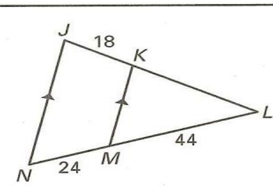


Example 2 *Solve a real-world problem*

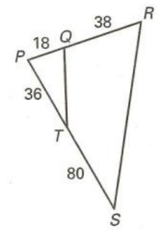
Aerodynamics A spoiler for a remote controlled car is shown where $AB = 31$ mm, $BC = 19$ mm, $CD = 27$ mm, and $DE = 23$ mm. *Explain* why $\overline{BD}$ is not parallel to $\overline{AE}$.



1. Find the length of $\overline{KL}$.



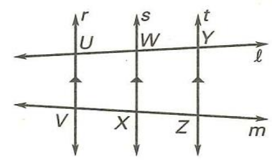
2. Determine whether $\overline{QT} \parallel \overline{RS}$.



THEOREM 6.6

If three parallel lines intersect two transversals, then they divide the transversals

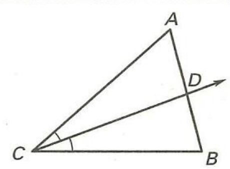
________________.



$\frac{UW}{WY} = $ ___

THEOREM 6.7

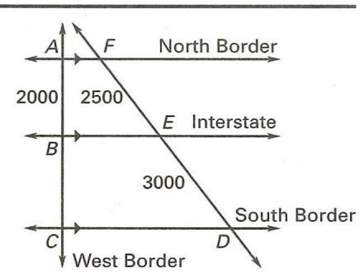
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are ________________ to the lengths of the other two sides.



$\frac{AD}{DB} = $ ___

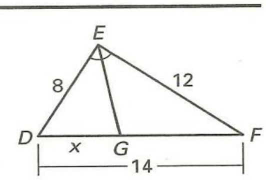
Example 3 *Use Theorem 6.6*

Farming A farmer's land is divided by a newly constructed interstate. The distances shown are in meters. Find the distance *CA* between the north border and the south border of the farmer's land.



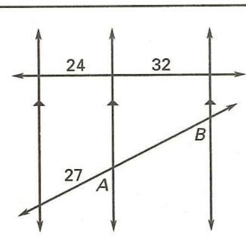
Example 4 *Use Theorem 6.7*

In the diagram, $\angle DEG \cong \angle GEF$. Use the given side lengths to find the length of $\overline{DG}$.



✔ ***Checkpoint*** Find the length of $\overline{AB}$.

3.



6.7 Perform Similarity Transformations

Dilation – a transformation that stretches or shrinks a figure to create a similar figure.

Scale factor – The scale factor (k) is the ratio of a side length of the image to the corresponding side length of the original figure.

Reduction – a dilation where $0<k<1$

Enlargement – a dilation where $k>1$

COORDINATE NOTATION FOR A DILATION

You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where k is the scale factor.

If $0 < k < 1$, the dilation is a __________. If $k > 1$, the dilation is an __________.

Example 1 *Draw a dilation with a scale factor greater than 1*

Draw a dilation of quadrilateral *ABCD* with vertices $A(2, 0)$, $B(6, -4)$, $C(8, 2)$, and $D(6, 4)$. Use a scale factor of $\frac{1}{2}$.

Example 2 *Verify that a figure is similar to its dilation*

A triangle has the vertices $A(2, -1)$, $B(4, -1)$, and $C(4, 2)$. The image of $\triangle ABC$ after a dilation with a scale factor of 2 is $\triangle DEF$.

a. Sketch $\triangle ABC$ and $\triangle DEF$.

b. Verify that $\triangle ABC$ and $\triangle DEF$ are similar.

Checkpoint **Complete the following exercises.**

1. A triangle has the vertices $B(-1, -1)$, $C(0, 1)$, and $D(1, 0)$. Find the coordinates of L, M, and N so that $\triangle LMN$ is a dilation of $\triangle BCD$ with a scale factor of 4. Sketch $\triangle BCD$ and $\triangle LMN$.