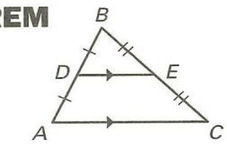
5.1 – Midsegment Theorem and Coordinate Proof

Midsegment of a triangle –

THEOREM 5.1: MIDSEGMENT THEOREM

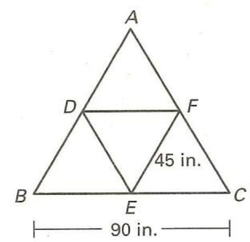
The segment connecting the midpoints of two sides of a triangle is ________ to the third side and is ______ as long as that side.



$\overline{DE} \parallel \overline{AC}$ and $DE = \frac{1}{2}AC$

Example 1 *Use the Midsegment Theorem to find lengths*

Windows A large triangular window is segmented as shown. In the diagram, $\overline{DF}$ and $\overline{EF}$ are midsegments of $\triangle ABC$. Find DF and AB.

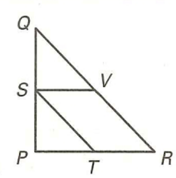


In the diagram for Example 1, midsegment $\overline{DF}$ can be called "the midsegment opposite $\overline{BC}$."

1. In Example 1, consider $\triangle ADF$. What is the length of the midsegment opposite $\overline{DF}$?

Example 2 *Use the Midsegment Theorem*

In the diagram at the right, $QS = SP$ and $PT = TR$. Show that $\overline{QR} \parallel \overline{ST}$.



Solution

Because $QS = SP$ and $PT = TR$, S is the __________ of $\overline{QP}$ and T is the __________ of $\overline{PR}$ by definition. Then $\overline{ST}$ is a __________ of $\triangle PQR$ by definition and $\overline{QR} \parallel \overline{ST}$ by the ____________________.

Checkpoint Complete the following exercise.

2. In Example 2, if V is the midpoint of $\overline{QR}$, what do you know about $\overline{SV}$?

5.2 – Use Perpendicular Bisectors

Perpendicular bisector –

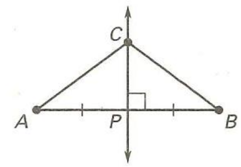
Equidistant –

Concurrent –

Point of concurrency –

THEOREM 5.2: PERPENDICULAR BISECTOR THEOREM

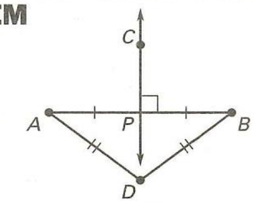
In a plane, if a point is on the perpendicular bisector of a segment, then it is ____________ from the endpoints of the segment.



If $\overleftrightarrow{CP}$ is the ⊥ bisector of $\overline{AB}$, then CA = ____.

THEOREM 5.3: CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM

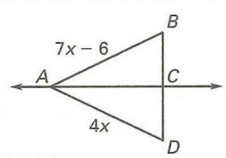
In a plane, if a point is equidistant from the endpoints of a segment, then it is on the ______________ ________ of the segment.



If $DA = DB$, then D lies on the __________ of $\overline{AB}$.

Example 1 *Use the Perpendicular Bisector Theorem*

$\overleftrightarrow{AC}$ is the perpendicular bisector of $\overline{BD}$. Find AD.



Solution

AD = ____ **Perpendicular Bisector Theorem**

____ = ______ **Substitute.**

x = ___ **Solve for *x*.**

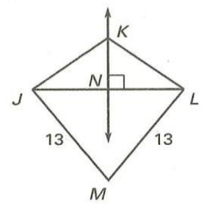
AD = ____ = _____ = ___.

Example 2 *Use perpendicular bisectors*

In the diagram, $\overleftrightarrow{KN}$ is the perpendicular bisector of $\overline{JL}$.

a. What segment lengths in the diagram are equal?

b. Is M on $\overleftrightarrow{KN}$?

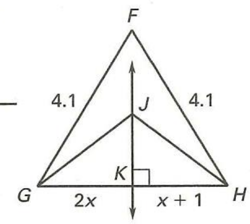


Checkpoint In the diagram, $\overleftrightarrow{JK}$ is the perpendicular bisector of $\overline{GH}$.

1. What segment lengths are equal?

____

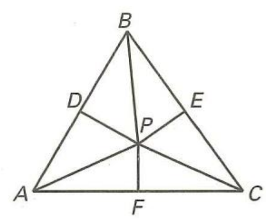
2. Find GH.



THEOREM 5.4: CONCURRENCY OF PERPENDICULAR BISECTORS OF A TRIANGLE

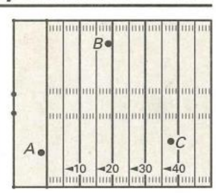
The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA =$ _____ $=$ _____.



Example 3 *Use the concurrency of perpendicular bisectors*

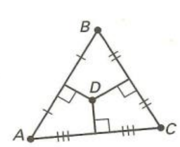
Football Three friends are playing catch. You want to join and position yourself so that you are the same distance from your friends. Find a location for you to stand.



Solution

Theorem 5.4 shows you that you can find a point equidistant from three points by using the ___ ___ ___ ___ of the triangle formed by those points.

Copy the positions of points A, B, and C and connect those points to draw $\triangle ABC$. Then use a ruler and a protractor to draw the three ___ ___ ___ ______ of $\triangle ABC$. The point of concurrency D is a location for you to stand.



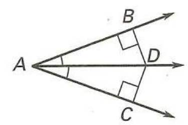
5.3 – Use Angle Bisectors of Triangles

Incenter –

> In Geometry, *distance* means the *shortest* length between two objects.

THEOREM 5.5: ANGLE BISECTOR THEOREM

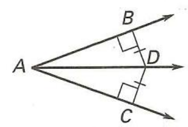
If a point is on the bisector of an angle, then it is equidistant from the two _______ of the angle.



If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB =$ _____.

THEOREM 5.6: CONVERSE OF THE ANGLE BISECTOR THEOREM

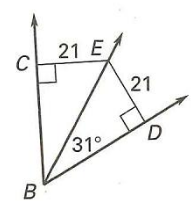
If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the __________ of the angle.



If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ ___ ___ ___ $\angle BAC$.

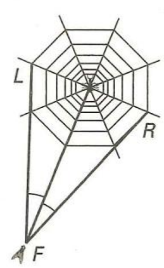
Example 1 ***Use the Angle Bisector Theorems***

Find the measure of $\angle CBE$.



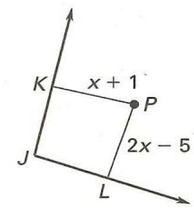
Example 2 ***Solve a real-world problem***

Web A spider's position on its web relative to an approaching fly and the opposite sides of the web forms congruent angles, as shown. Will the spider have to move farther to reach a fly toward the right edge or the left edge?



Example 3 *Use algebra to solve a problem*

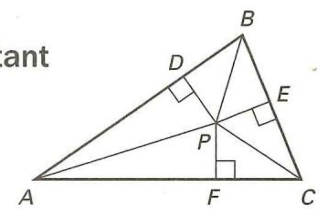
For what value of x does P lie on the bisector of $\angle J$?



THEOREM 5.7: CONCURRENCY OF ANGLE BISECTORS OF A TRIANGLE

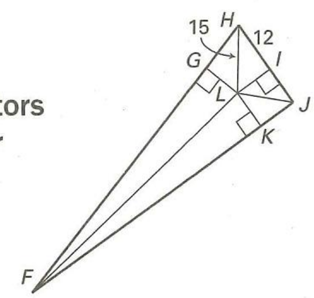
The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD =$ ____ $=$ ____.



Example 4 *Use the concurrency of angle bisectors*

In the diagram, L is the incenter of $\triangle FHJ$. Find LK.



By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter L is ____________ from the sides of $\triangle FHJ$. So, to find LK, you can find ___ in $\triangle LHI$. Use the Pythagorean Theorem.

____ = ______		**Pythagorean Theorem**
____ = ________		**Substitute known values.**
____ = _____		**Simplify.**
__ = __		**Take the positive square root of each side.**

Because ___ $= LK$, $LK =$ ___.

5.4 – Use Medians and Altitudes

VOCABULARY

Median of a triangle The median of a triangle is a segment from a vertex to the midpoint of the opposite side.

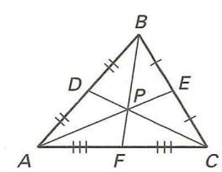
Centroid The point of concurrency of the three medians of a triangle is the centroid.

Altitude of a triangle An altitude of a triangle is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.

Orthocenter The point at which the lines containing the three altitudes of a triangle intersect is called the orthocenter of the triangle.

THEOREM 5.8: CONCURRENCY OF MEDIANS OF A TRIANGLE

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

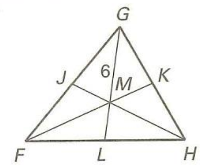


The medians of $\triangle ABC$ meet at P and $AP = \frac{2}{3}$_____,

$BP = \frac{2}{3}$_____, and $CP = \frac{2}{3}$_____.

Example 1 *Use the centroid of a triangle*

In △*FGH*, *M* is the centroid and *GM* = 6. Find *ML* and *GL*.



___ = $\frac{\_\_}{\_\_}$ *GL* — Concurrency of Medians of a Triangle Theorem

___ = $\frac{\_\_}{\_\_}$ *GL* — Substitute ___ for *GM*.

___ = *GL* — Multiply each side by the reciprocal, $\frac{\_\_}{\_\_}$.

Then *ML* = *GL* − ______ = ___ − ___ = ___.

So, *ML* = ___ and *GL* = ___.

Checkpoint Complete the following exercise.

1. In Example 1, suppose *FM* = 10. Find *MK* and *FK*.

Example 2 *Find the centroid of a triangle*

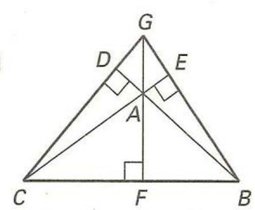
The vertices of △*JKL* are *J*(1, 2), *K*(4, 6), and *L*(7, 4). Find the coordinates of the centroid *P* of △*JKL*.

Sketch △*JKL*. Then use the Midpoint Formula to find the midpoint *M* of $\overline{JL}$ and sketch median $\overline{KM}$.

THEOREM 5.9: CONCURRENCY OF ALTITUDES OF A TRIANGLE

The lines containing the altitudes of a triangle are ____________.

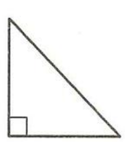
The lines containing $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ meet at *G*.



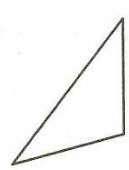
Example 3 *Find the orthocenter*

Find the orthocenter *P* in the triangle.

a.



b.

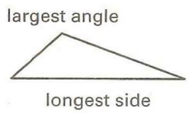


5.5 – Use Inequalities in a Triangle

Example 1 ***Relate side length and angle measure***

Mark the largest angle, longest side, smallest angle, and shortest side of the triangle shown at the right. What do you notice?

Solution

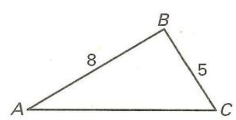


The longest side and largest angle are ________ each other.

The shortest side and smallest angle are ________ each other.

THEOREM 5.10

If one side of a triangle is longer than another side, then the angle opposite the longer side is ________ than the angle opposite the shorter side.

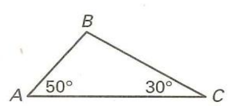


AB > *BC*, so

$m\angle$ ____ > $m\angle$ ____.

THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is ________ than the side opposite the smaller angle.



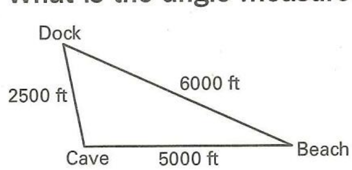
$m\angle A > m\angle C$,

so ____ > ____.

> Be careful not to confuse the symbol ∠ meaning *angle* with the symbol < meaning *is less than*. Notice that the bottom edge of the angle symbol is horizontal.

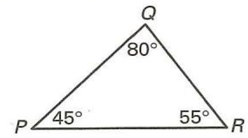
Example 2 ***Find angle measures***

Boating A long-tailed boat leaves a dock and travels 2500 feet to a cave, 5000 feet to a beach, then 6000 feet back to the dock as shown below. One of the angles in the path is about 55° and one is about 24°. What is the angle measure of the path made at the cave?



Checkpoint Complete the following exercises.

1. List the sides of $\triangle PQR$ in order from shortest to longest.



2. Another boat makes a trip whose path has sides of 1.5 miles, 2 miles, and 2.5 miles long and angles of 90°, about 53°, and about 37°. Sketch and label a diagram with the shortest side on the bottom and the right angle at the right.

THEOREM 5.12: TRIANGLE INEQUALITY THEOREM

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

_____ + _____ > AC

AC + _____ > _____

_____ + AC > _____

Example 3 *Find possible side lengths*

A triangle has one side of length 14 and another of length 10. Describe the possible lengths of the third side.

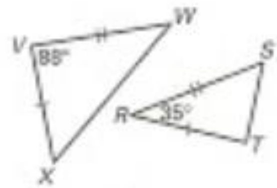
Solution

Let x represent the length of the third side. Draw diagrams to help visualize the small and large values of x. Then use the Triangle Inequality Theorem to write and solve inequalities.

5.6 –Inequalities in Two Triangles and Indirect Proof

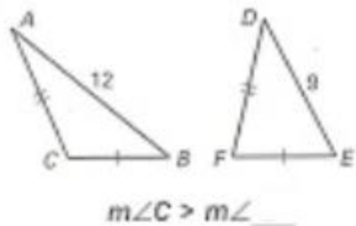
THEOREM 5.13: HINGE THEOREM

If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is ________ than the third side of the second.



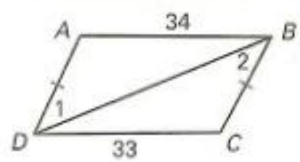
THEOREM 5.14: CONVERSE OF THE HINGE THEOREM

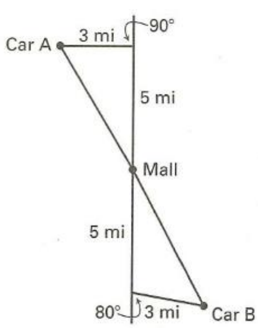
If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is ________ than the included angle of the second.



Example 1 *Use the Converse of the Hinge Theorem*

Given that $\overline{AD} \cong \overline{BC}$, how does $\angle 1$ compare to $\angle 2$?





Example 2 *Solve a multi-step problem*

Travel Car A leaves a mall, heads due north for 5 mi and then turns due west for 3 mi. Car B leaves the same mall, heads due south for 5 mi and then turns 80° toward east for 3 mi. Which car is farther from the mall?