4. 1 – Apply Triangle Sum Properties

Triangle –

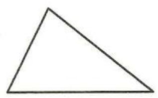
Interior angles –

Exterior angles –

Corollary to a theorem –

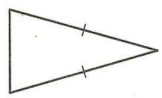
CLASSIFYING TRIANGLES BY SIDES

Scalene Triangle



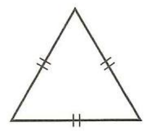
_____ congruent sides

Isosceles Triangle



At least ___ congruent sides

Equilateral Triangle

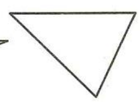


___ congruent sides

CLASSIFYING TRIANGLES BY ANGLES

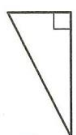
Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

Acute Triangle



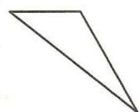
___ acute angles

Right Triangle



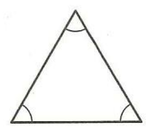
___ right angle

Obtuse Triangle



___ obtuse angle

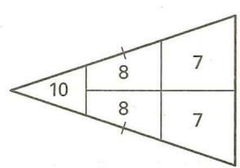
Equiangular Triangle



___ congruent angles

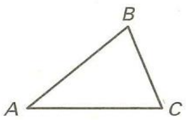
Example 1 ***Classify triangles by sides and by angles***

Shuffleboard Classify the triangular shape of the shuffleboard scoring area in the diagram by its sides and by measuring its angles.



THEOREM 4.1: TRIANGLE SUM THEOREM

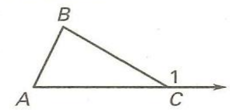
The sum of the measures of the interior angles of a triangle is _______.



$m\angle A + m\angle B + m\angle C =$ _______

THEOREM 4.2: EXTERIOR ANGLE THEOREM

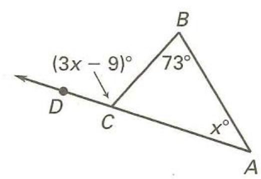
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two _______________ angles.



$m\angle 1 = m\angle$___ $+ m\angle$___

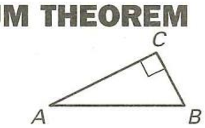
Example 3 *Find angle measure*

Use the diagram at the right to find the measure of $\angle DCB$.



COROLLARY TO THE TRIANGLE SUM THEOREM

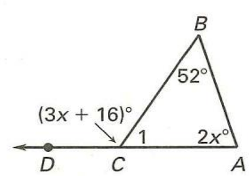
The acute angles of a right triangle are _______________.



$m\angle A + m\angle B =$ _______

2. Triangle *JKL* has vertices *J*(−2, −1), *K*(1, 3), and *L*(5, 0). Classify it by its sides. Then determine if it is a right triangle.

3. Find the measure of $\angle 1$ in the diagram shown.

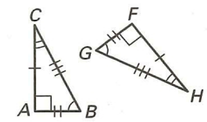


4.2 – Apply Congruence and Triangles

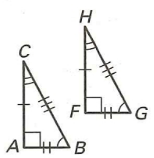
Congruent figures –

Example 1 *Identify congruent parts*

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



To help you identify corresponding parts, turn $\triangle FGH$.



Solution

The diagram indicates that $\triangle ABC \cong \triangle$______.

Corresponding angles $\angle A \cong$ _____, $\angle B \cong$ _____, $\angle C \cong$ _____

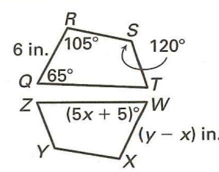
Corresponding sides $\overline{AB} \cong$ _____, $\overline{BC} \cong$ _____, $\overline{CA} \cong$ _____

Example 2 *Use properties of congruent figures*

In the diagram, $QRST \cong WXYZ$.

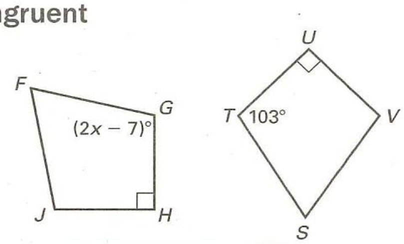
a. Find the value of x.

b. Find the value of y.



Solution

1. Identify all pairs of congruent corresponding parts.



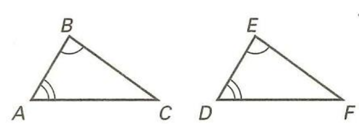
Corresponding angles:

Corresponding sides.

2. Find the value of x and find $m\angle G$.

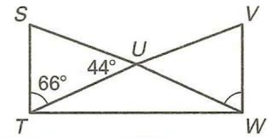
THEOREM 4.3: THIRD ANGLES THEOREM

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also __________.



Example 4 *Use the Third Angles Theorem*

Find $m\angle V$.

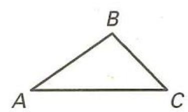


$\angle SUT \cong \angle VUW$ by the ________________ ________________.
The diagram shows that $\angle STU \cong$ __________, so by the Third Angles Theorem, $\angle S \cong$ _____. By the Triangle Sum Theorem, $m\angle S =$ ____________________ $=$ _____. So, $m\angle S = m\angle V =$ _____ by the definition of congruent angles.

THEOREM 4.4: PROPERTIES OF CONGRUENT TRIANGLES

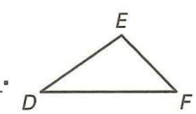
Reflexive Property of Congruent Triangles

For any triangle ABC, $\triangle ABC \cong$ __________.



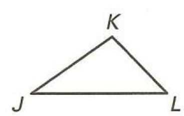
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then ________ ______________.



Transitive Property of Congruent Triangles

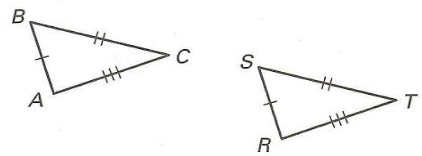
If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then ________________ ________.



4.3 – Prove Triangles Congruent by SSS

POSTULATE 19: SIDE-SIDE-SIDE (SSS) CONGRUENCE POSTULATE

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.



If Side $\overline{AB} \cong$ ______,

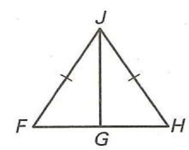
Side $\overline{BC} \cong$ ______, and

Side $\overline{CA} \cong$ ______,

then $\triangle ABC \cong$ ________.

Example 1 ***Use the SSS Congruence Postulate***

Write a proof.

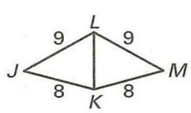


Given $\overline{FJ} \cong \overline{HJ}$,
G is the midpoint of $\overline{FH}$.

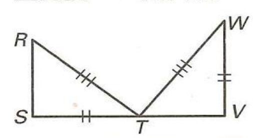
Prove $\triangle FGJ \cong \triangle HGJ$

Proof It is given that $\overline{FJ} \cong$ _____. Point G is the midpoint of $\overline{FH}$, so __________. By the Reflexive Property, __________. So, by the ______________________________, $\triangle FGJ \cong \triangle HGJ$.

1. $\triangle JKL \cong \triangle MKL$

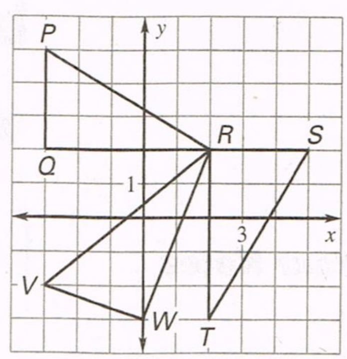


2. $\triangle RST \cong \triangle TVW$



Example 2 ***Congruence in the coordinate plane***

Determine whether $\triangle PQR$ is congruent to the other triangles shown at the right.



4.4 – Prove Triangles Congruent by SAS and HL

Hypotenuse –

Leg of a right triangle –

POSTULATE 20: SIDE-ANGLE-SIDE (SAS) CONGRUENCE POSTULATE

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If Side $\overline{RS} \cong$ ____,

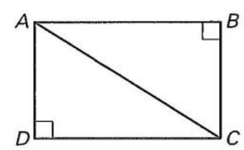
Angle $\angle R \cong$ ____, and

Side $\overline{RT} \cong$ ____,

then $\triangle RST \cong$ ____.

Example 2 *Use SAS and properties of shapes*

In the diagram, *ABCD* is a rectangle. What can you conclude about $\triangle ABC$ and $\triangle CDA$?

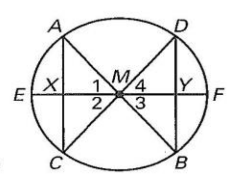


Solution

By the Right Angles Congruence Theorem,

$\angle B \cong \angle D$. Opposite sides of a rectangle are congruent, so ____ and ____.

$\triangle ABC$ and $\triangle CDA$ are congruent by the ____.

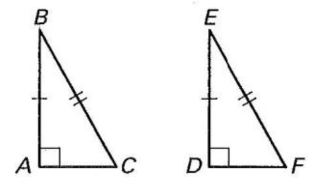


Checkpoint **In the diagram, $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ pass through the center *M* of the circle. Also, $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$.**

1. Prove that $\triangle DMY \cong \triangle BMY$.

THEOREM 4.5: HYPOTENUSE-LEG CONGRUENCE THEOREM

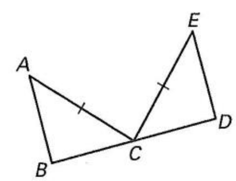
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second triangle, then the two triangles are ______ ______.



Example 3 ***Use the Hypotenuse-Leg Theorem***

Write a proof.

Given $\overline{AC} \cong \overline{EC}$,
$\overline{AB} \perp \overline{BD}$,
$\overline{ED} \perp \overline{BD}$,
$\overline{AC}$ is a bisector of $\overline{BD}$.



Prove $\triangle ABC \cong \triangle EDC$

	Statements	Reasons
H	1. $\overline{AC} \cong \overline{EC}$	1. ______
	2. $\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{BD}$	2. ______
	3. $\angle B$ and $\angle D$ are ______ ______.	3. Definition of $\perp$ lines
	4. $\triangle ABC$ and $\triangle EDC$ are ______ ______.	4. Definition of a ______ ______
	5. $\overline{AC}$ is a bisector of $\overline{BD}$.	5. ______
L	6. $\overline{BC} \cong \overline{DC}$	6. Definition of segment bisector
	7. $\triangle ABC \cong \triangle EDC$	7. ______ ______ ______

4.5 – Prove Triangles Congruent by ASA

POSTULATE 21: ANGLE-SIDE-ANGLE (ASA) CONGRUENCE POSTULATE

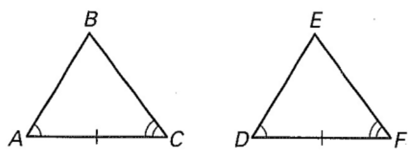
If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If Angle $\angle A \cong$ _____,

Side $\overline{AC} \cong$ _____, and

Angle $\angle C \cong$ _____,

then $\triangle ABC \cong$ _______.



THEOREM 4.6: ANGLE-ANGLE-SIDE (AAS) CONGRUENCE THEOREM

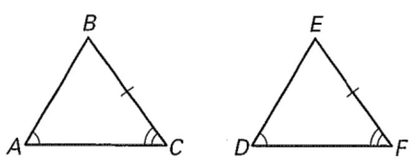
If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

If Angle $\angle A \cong$ _____,

Angle $\angle C \cong$ _____, and

Side $\overline{BC} \cong$ _____,

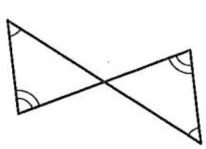
then $\triangle ABC \cong$ _______.



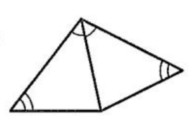
Example 1 ***Identify congruent triangles***

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

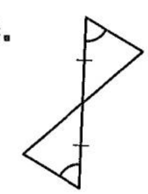
a.



b.

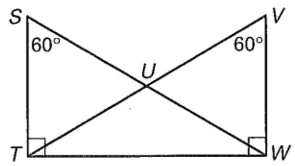


c.

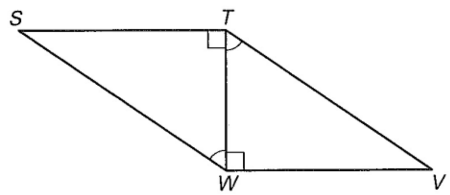


✔ *Checkpoint* **Can $\triangle STW$ and $\triangle VWT$ be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.**

1.



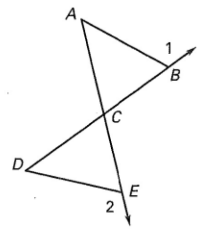
2.



4.6 – Use Congruent Triangles

Example 1 ***Use congruent triangles***

Explain **how you can use the given information to prove that the triangles are congruent.**



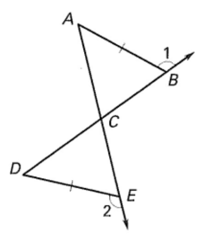
Given $\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{DE}$

Prove $\overline{DC} \cong \overline{AC}$

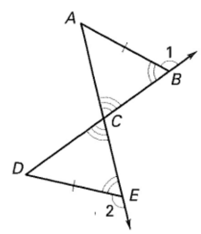
Solution

If you can show that ________________, you will know that $\overline{DC} \cong \overline{AC}$. First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, $\angle ABC$ and $\angle DEC$ are ___ __________ to congruent angles, so $\angle$______ $\cong \angle$______. Also, $\angle ACB \cong$ ___ ___.

Mark given information.

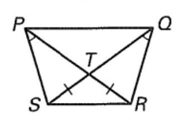


Add deduced information.



Two angle pairs and a ______________ side are congruent, so by the ______ Congruence Theorem, $\triangle ABC \cong \triangle DEC$. Because ___ __________________ of congruent triangles are congruent, $\overline{DC} \cong \overline{AC}$.

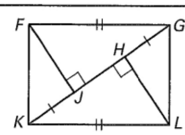
1. ***Explain*** how you can prove that $\overline{PR} \cong \overline{QS}$.



Checkpoint Use the given information to write a plan for proof.

3. Given $\overline{GH} \cong \overline{KJ}$, $\overline{FG} \cong \overline{LK}$, $\angle FJG$ and $\angle LHK$ are rt. $\angle$s.

Prove $\triangle FJK \cong \triangle LHG$



4.7 – Use Isosceles and Equilateral Triangles

Legs –

Vertex angle –

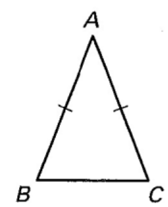
Base –

Base angles –

THEOREM 4.7: BASE ANGLES THEOREM

If two sides of a triangle are congruent, then the angles opposite them are congruent.

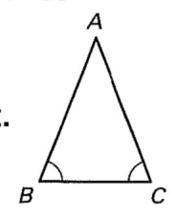
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong$ ____.



THEOREM 4.8: CONVERSE OF BASE ANGLES THEOREM

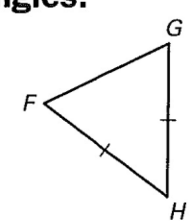
If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong$ ____.



Example 1 ***Apply the Base Angles Theorem***

In $\triangle FGH$, $\overline{FH} \cong \overline{GH}$. Name two congruent angles.



Solution

$\overline{FH} \cong \overline{GH}$, so by the Base Angles Theorem,

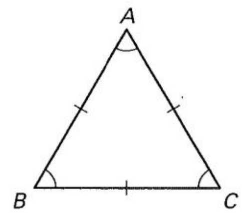
> The corollaries state that a triangle is *equilateral* if and only if it is *equiangular*.

COROLLARY TO THE BASE ANGLES THEOREM

If a triangle is equilateral, then it is ______________.

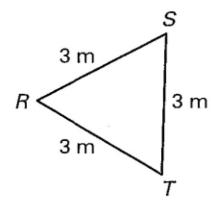
COROLLARY TO THE CONVERSE OF BASE ANGLES THEOREM

If a triangle is equiangular, then it is ______________.



Example 2 ***Find measures in a triangle***

Find the measures of $\angle R$, $\angle S$, and $\angle T$.



Solution

The diagram shows that $\triangle RST$ is ______________. Therefore, by the Corollary to the Base Angles Theorem, $\triangle RST$ is ______________. So, $m\angle R = m\angle S = m\angle T$.

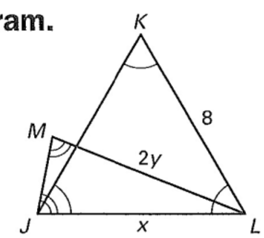
$3(m\angle R) =$ ______ **Triangle Sum Theorem**

$m\angle R =$ ______ **Divide each side by 3.**

The measures of $\angle R$, $\angle S$, and $\angle T$ are all ____.

Example 3 ***Use isosceles and equilateral triangles***

Find the values of *x* and *y* in the diagram.



> You cannot use $\angle J$ to refer to $\angle LJM$ because three angles have *J* as their vertex.

Solution

Step 1 Find the value of *x*. Because $\triangle JKL$ is ______________, it is also ______________ and $\overline{KL} \cong$ ____. Therefore, $x =$ ____.

Step 2 Find the value of *y*. Because $\angle JML \cong$ ______, $\overline{LM} \cong$ ____, and $\triangle LMJ$ is isosceles. You know that $LJ =$ ____.

$LM =$ ____ **Definition of congruent segments**

$2y =$ ____ **Substitute 2*y* for *LM* and ____ for *LJ*.**

$y =$ ____ **Divide each side by 2.**

4.8 – Use Isosceles and Equilateral Triangles

Transformation – an operation that moves or changes a geometric figure in some way to produce a new figure

Image – the new figure produced by a transformation is the image

Translation – moves every point of a figure the same distance in the same direction

Reflection – uses a line of reflection to create a mirror image of the original figure

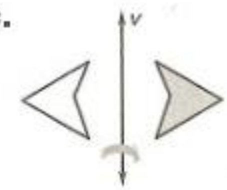
Example 1 *Identify transformations*

Name the type of transformation demonstrated in each picture.

a.

b.

c.



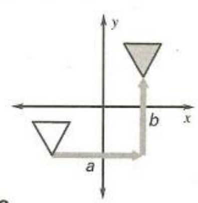
Your Notes

COORDINATE NOTATION FOR A TRANSLATION

You can describe a translation by the notation

$(x, y) \rightarrow (x + a, y + b)$

which shows that each point (x, y) of the unshaded figure is translated horizontally a units and vertically b units.



Example 2 *Translate a figure in the coordinate plane*

Figure *ABCD* has the vertices $A(1, 2)$, $B(3, 3)$, $C(4, -1)$, and $D(1, -2)$. Sketch *ABCD* and its image after the translation $(x, y) \rightarrow (x - 4, y + 2)$.

Your Notes

Example 3 *Reflect a figure in the x-axis*

Shapes You are cutting figures out of paper. Use a reflection in the x-axis to draw the other half of the figure.