3.1 – Lines and Angles

Parallel lines –

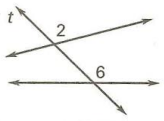
Skew lines –

Parallel planes –

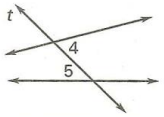
Transversal –

ANGLES FORMED BY TRANSVERSALS

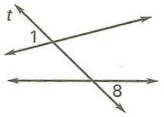
Two angles are _____ angles if they have corresponding positions. For example, $\angle 2$ and $\angle 6$ are above the lines and to the right of the transversal t.



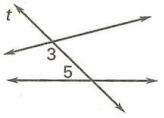
Two angles are _____ _____ angles if they lie between the two lines and on opposite sides of the transversal.



Two angles are _____ _____ angles if they lie outside the two lines and on opposite sides of the transversal.



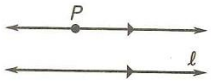
Two angles are _____ _____ angles if they lie between the two lines and on the same side of the transversal.



Another name for consecutive interior angles is same-side interior angles.

POSTULATE 13 PARALLEL POSTULATE

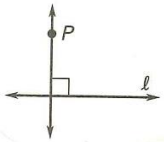
If there is a line and a point not on the line, then there is _____ _____ line through the point parallel to the given line.



There is exactly one line through P parallel to ℓ.

POSTULATE 14 PERPENDICULAR POSTULATE

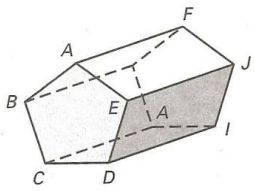
If there is a line and a point not on the line, then there is _____ line through the point perpendicular to the given line.



There is exactly one line through P perpendicular to ℓ.

Example 1 *Identify relationships in space*

Think of each segment in the figure as part of a line. Which line(s) or plane(s) in the figure appear to fit the description?



a. Line(s) parallel to $\overleftrightarrow{AF}$ and containing point *E*

b. Line(s) skew to $\overleftrightarrow{AF}$ and containing point *E*

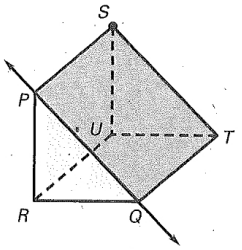
c. Line(s) perpendicular to $\overleftrightarrow{AF}$ and containing point *E*

d. Plane(s) parallel to plane *FGH* and containing point *E*

1. parallel to $\overleftrightarrow{PQ}$ and contains *S*

2. perpendicular to $\overleftrightarrow{PQ}$ and contains *S*

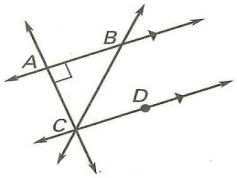
3. skew to $\overleftrightarrow{PQ}$ and contains *S*



4. Name the plane that contains *S* and appears to be parallel to plane *PQR*.

Example 2 *Identify parallel and perpendicular lines*

Use the diagram at the right to answer each question.



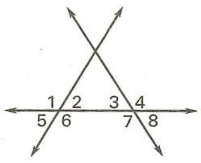
a. Name a pair of parallel lines.

b. Name a pair of perpendicular lines.

c. Is $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$? *Explain.*

Example 3 *Identify angle relationships*

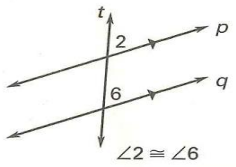
Identify all pairs of (a) corresponding angles, (b) alternate interior angles, (c) alternate exterior angles, and (d) consecutive interior angles.



3.2 – Use Parallel Lines and Transversals

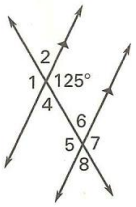
POSTULATE 15 CORRESPONDING ANGLES POSTULATE

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are _____.



Example 1 *Identify congruent angles*

The measure of three of the numbered angles is 125°. Identify the angles. *Explain* your reasoning.



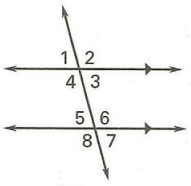
Solution

By the Corresponding Angles Postulate, _____ = 125°.

Using the Vertical Angles Congruence Theorem, _____ = 125°.

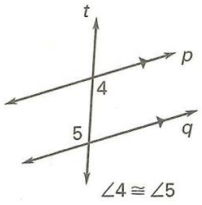
Because ∠1 and ∠5 are corresponding angles, by the _____, you know that _____ = 125°.

1. If $m\angle 7 = 75°$, find $m\angle 1$, $m\angle 3$, and $m\angle 5$. Tell which postulate or theorem you use in each case.



THEOREM 3.1 ALTERNATE INTERIOR ANGLES THEOREM

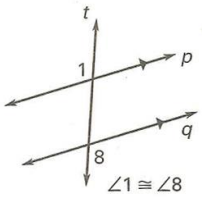
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are _____.



$\angle 4 \cong \angle 5$

THEOREM 3.2 ALTERNATE EXTERIOR ANGLES THEOREM

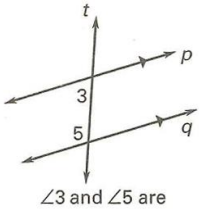
If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are _____.



$\angle 1 \cong \angle 8$

THEOREM 3.3 CONSECUTIVE INTERIOR ANGLES THEOREM

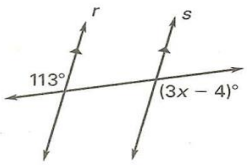
If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are _____ _____.



$\angle 3$ and $\angle 5$ are supplementary.

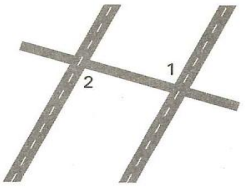
Example 2 ***Use properties of parallel lines***

Find the value of x.

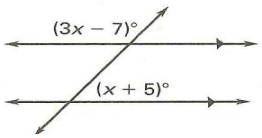


Example 3 ***Solve a real-world problem***

Runways A taxiway is being constructed that intersects two parallel runways at an airport. You know that $m\angle 2 = 98°$. What is $m\angle 1$? How do you know?



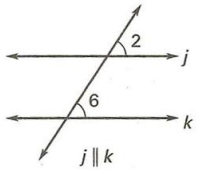
2. Find the value of x.



3.3 – Prove Lines are Parallel

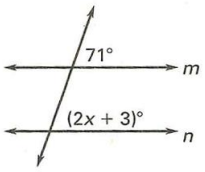
POSTULATE 16 CORRESPONDING ANGLES CONVERSE

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are _____.



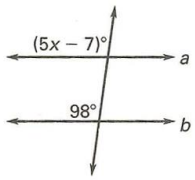
Example 1 ***Apply the Corresponding Angles Converse***

Find the value of x that makes $m \parallel n$.



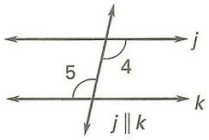
Checkpoint **Find the value of x that makes $a \parallel b$.**

1.



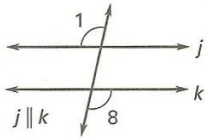
THEOREM 3.4 ALTERNATE INTERIOR ANGLES CONVERSE

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are _____.



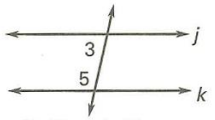
THEOREM 3.5 ALTERNATE EXTERIOR ANGLES CONVERSE

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are _____.



THEOREM 3.6 CONSECUTIVE INTERIOR ANGLES CONVERSE

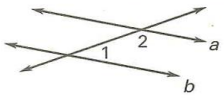
If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are _____.



If ∠3 and ∠5 are supplementary, then $j \parallel k$.

Checkpoint Can you prove that lines a and b are parallel? *Explain* why or why not.

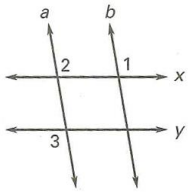
2. $m\angle 1 + m\angle 2 = 180°$



Your Notes

Example 3 *Write a paragraph proof*

In the figure, $a \parallel b$ and $\angle 1$ is congruent to $\angle 3$. Prove $x \parallel y$.

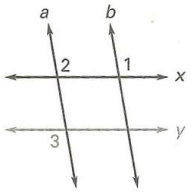


Solution

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

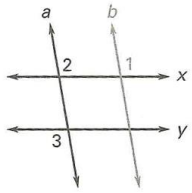
Plan for Proof

a. Look at $\angle 1$ and $\angle 2$.



_____ because $a \parallel b$.

b. Look at $\angle 2$ and $\angle 3$.



If $\angle 2 \cong \angle 3$ then _____.

In paragraph proofs, transitional words such as *so, then,* and *therefore* help to make the logic clear.

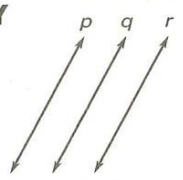
Plan in Action

a. It is given that $a \parallel b$, so by the _____ _____, $\angle 1 \cong \angle 2$.

b. It is also given that $\angle 1 \cong \angle 3$. Then _____ by the Transitive Property of Congruence for angles. Therefore, by the _____ _____ _____ _____, $x \parallel y$.

THEOREM 3.7 TRANSITIVE PROPERTY OF PARALLEL LINES

If two lines are parallel to the same line, then they are _____ to each other.



If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

3.4 – Find and Use Slopes of Lines

Slope -

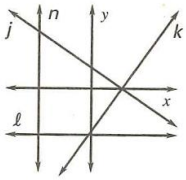
SLOPE OF LINES IN THE COORDINATE PLANE

Negative slope: _____ from left to right, as in line *j*

Positive slope: _____ from left to right, as in line *k*

Undefined slope: _____, as in line *n*

Zero slope (slope of 0): _____, as in line *ℓ*



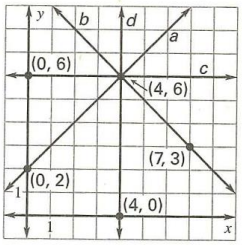
Slope

$m = \frac{\text{rise}}{\text{run}}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

Example 1 ***Find slopes of lines in a coordinate plane***

Find the slope of line *a* and line *c*.



Slope of line *a*:

$m =$

Slope of line *c*:

$m =$

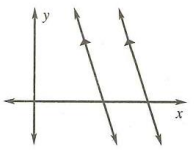
Checkpoint **Use the graph in Example 1. Find the slope of the line.**

1. line *b*	2. line *d*

POSTULATE 17 SLOPES OF PARALLEL LINES

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same _____.

Any two _____ lines are parallel.



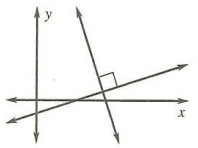
$m_1 = m_2$

If the product of two numbers is −1, then the numbers are called *negative reciprocals*.

POSTULATE 18 SLOPES OF PERPENDICULAR LINES

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is _____.

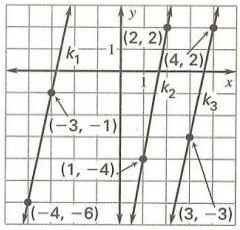
Horizontal lines are _____ to vertical lines.



$m_1 \cdot m_2 = -1$

Example 2 *Identify parallel lines*

Find the slope of each line. Which lines are parallel?



Solution

Find the slope of k_1.

$m =$

Find the slope of k_2.

$m =$

Find the slope of k_3.

$m =$

3. Line c passes through $(2, -2)$ and $(5, 7)$. Line d passes through $(-3, 4)$ and $(1, -8)$. Are the two lines parallel? *Explain* how you know.

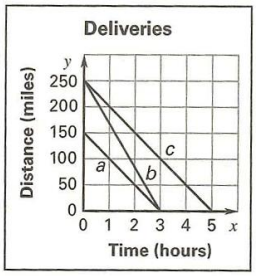
Example 3 *Draw a perpendicular line*

Line h passes through $(1, -2)$ and $(5, 6)$. Graph the line perpendicular to h that passes through the point $(2, 5)$.

4. Line n passes through $(1, 6)$ and $(8, 4)$. Line m passes through $(0, 5)$ and $(2, 12)$. Is $n \perp m$? *Explain*.

Example 4 *Analyze graphs*

Delivery A trucker made three deliveries. The graph shows the trucker's distance to the destination from the starting time to the arrival time for each delivery. Use slopes to make a statement about the deliveries.



The rate at which the trucker drives is represented by the _____ of the segments. Segments ___ and ___ have the same slope, so deliveries a and c were driven at the same _____.

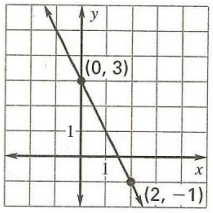
3.5 – Write and Graph Equations of Lines

Slope-intercept form –

Standard form –

Example 1 *Write an equation of a line from a graph*

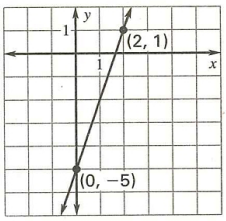
Write an equation of the line in slope-intercept form.



Example 2 *Write an equation of a parallel line*

Write an equation of the line passing through the point (1, −1) that is parallel to the line with the equation $y = 2x - 1$.

1. Write an equation of the line in the graph at the right.



2. Write an equation of the line that passes through the point $(-2, 5)$ and is parallel to the line with the equation $y = -2x + 3$.

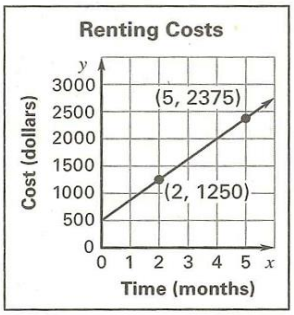
Example 3 *Write an equation of a perpendicular line*

Write an equation of the line *j* passing through the point (3, 2) that is perpendicular to the line *k* with the equation $y = -3x + 1$.

3. Write an equation of the line passing through the point $(-8, -2)$ that is perpendicular to the line with the equation $y = 4x - 3$.

Example 4 ***Write an equation of a line from a graph***

Rent The graph models the total cost of renting an apartment. Write an equation of the line. *Explain* the meaning of the slope and the y-intercept of the line.



Example 6 ***Solve a real-world problem***

Subscriptions You can buy a magazine at a store for $3. You can subscribe yearly to the magazine for a flat fee of $18. After how many magazines is the subscription a better buy?

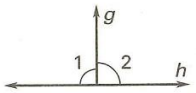
3.6 – Prove Theorems about Perpendicular Lines

Distance from a point to a line –

THEOREM 3.8

If two lines intersect to form a linear pair of congruent angles, then the lines are _____.

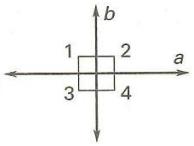
If $\angle 1 \cong \angle 2$, then g ___ h.



THEOREM 3.9

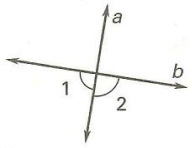
If two lines are perpendicular, then they intersect to form four _____ ___.

If $a \perp b$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are _____.



Example 1 *Draw conclusions*

In the diagram at the right, $\angle 1 \cong \angle 2$. What can you conclude about a and b?



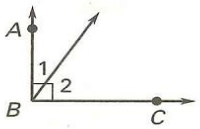
Solution

Lines a and b intersect to form a _____ _____, $\angle 1$ and $\angle 2$. So, by Theorem 3.8, _____.

THEOREM 3.10

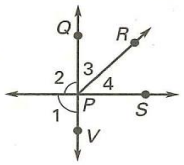
If two sides of two adjacent acute angles are perpendicular, then the angles are _____.

If $\overrightarrow{BA} \perp \overrightarrow{BC}$, then $\angle 1$ and $\angle 2$ are _____.



Example 2 ***Write a proof***

In the diagram at the right, $\angle 1 \cong \angle 2$. Prove that $\angle 3$ and $\angle 4$ are complementary.

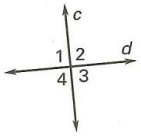


Given $\angle 1 \cong \angle 2$

Prove $\angle 3$ and $\angle 4$ are complementary.

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. _____
2. _____	2. Theorem 3.8
3. $\angle 3$ and $\angle 4$ are complementary.	3. _____

1. If $c \perp d$, what do you know about the sum of the measures of $\angle 3$ and $\angle 4$? *Explain*.



2. Using the diagram in Example 2, complete the following proof that $\angle QPS$ and $\angle 1$ are right angles.

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. _____
2. $\overleftrightarrow{PS} \perp \overleftrightarrow{PQ}$	2. _____
3. $\angle QPS$ and $\angle 1$ are right angles.	3. _____