2.1–Use Inductive Reasoning

- **Conjecture –**

- **Inductive reasoning –**

- **Counterexample –**

Example 1 *Describe a visual pattern*

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Figure 1

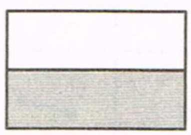



Figure 2

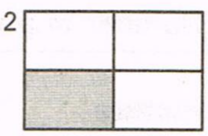
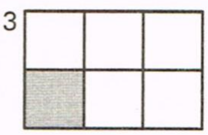



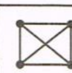
Figure 3


1. Sketch the fifth figure in the pattern in Example 1.

Example 3 *Make a conjecture*

Given five noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Make a table and look for a pattern. Notice the pattern in how the number of connections __________. You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture					
Number of connections	___	___	___	___	?

+___ +___ +___ +?

Conjecture You can connect five noncollinear points ______, or ____ different ways.

2. Describe the pattern in the numbers 1, 2.5, 4, 5.5, . . . and write the next three numbers in the pattern.

Example 4 *Make and test a conjecture*

Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of any three consecutive odd numbers.

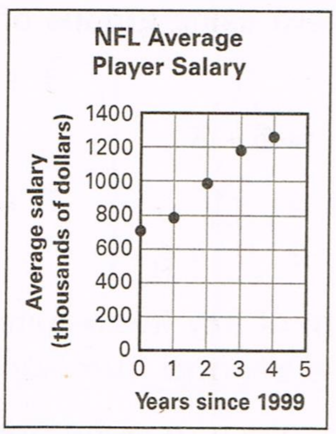
Example 5 *Find a counterexample*

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The difference of any two numbers is always smaller than the larger number.

Example 6 *Making conjectures from data displays*

The scatter plot shows the average salary of players in the National Football League (NFL) since 1999. Make a conjecture based on the graph.



5. Find a counterexample to show that the following conjecture is false.

Conjecture The quotient of two numbers is always smaller than the dividend.

2.2 – Analyze Conditional Statements

- **Conditional statement**

- **If-then form**

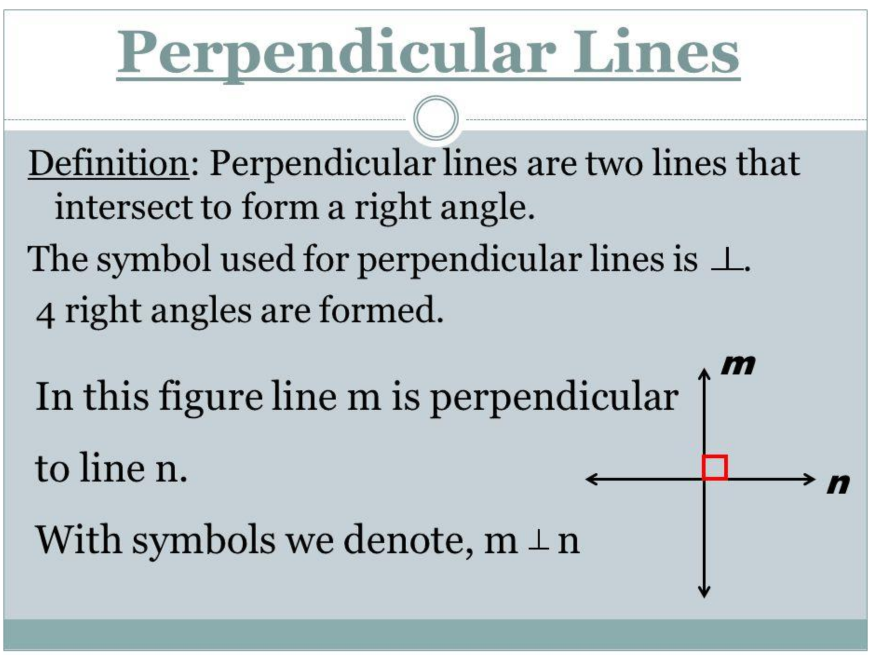
- **Negation**

- **Conditional statement**
  - **Converse**
  - **Inverse**
  - **Contrapositive**

- **Equivalent Statements**

- **Biconditional statements**

- **Perpendicular Lines (Definition)**



Example 1 ***Rewriting in If-Then Form***

Rewrite the conditional statement in *if-then* form.

a. Three points are coplanar if they lie on the same plane.

b. Water freezes at temperatures below 32°F.

c. An even number is divisible by 2.

Example 2 ***Writing an Inverse, Converse, and Contrapositive***

Write the (a) inverse, (b) converse, and (c) contrapositive of the following statement.

If the sun is shining, then we are not watching TV.

Solution

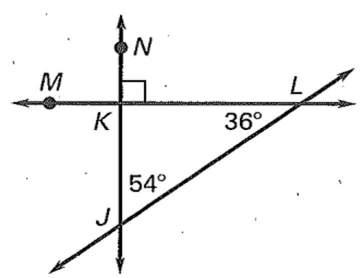
a. Inverse: ____.

b. Converse: ____.

c. Contrapositive: ____

Example 1 ***Using Definitions***

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a. $\angle KLJ$ and $\angle KJL$ are complementary.

b. $\overleftrightarrow{KL}$ and $\overleftrightarrow{LJ}$ are perpendicular.

c. $\angle MKJ$ is a right angle.

Example 2 ***Rewriting a Biconditional Statement***

Rewrite the following biconditional statement as a conditional statement and its converse.

An angle is a straight angle if and only if its measure is 180°.

Example 3 ***Analyzing a Biconditional Statement***

Consider the following statement: $x = 2$ if and only if $3x + 5x = 10x - 2x$.

a. Is this a biconditional statement? **b.** Is the statement true?

2.3 – Apply Deductive Reasoning

Deductive reasoning –

LAW OF DETACHMENT

If $p \rightarrow q$ is a true conditional statement and p is true, then ___________.

LAW OF SYLLOGISM

If $p \rightarrow q$ and $q \rightarrow r$ are true conditional statements, then ______________.

Example 3 ***Using the Law of Detachment***

State whether the argument is valid.

a. If Roger gets a part-time job, then he will buy a new bicycle. Roger buys a new bicycle. So, Roger got a part-time job.

b. If two angles are vertical angles, then they are congruent. $\angle 1$ and $\angle 2$ are vertical angles. So, $\angle 1$ and $\angle 2$ are congruent.

5. State whether the following argument is valid. If two adjacent angles form a straight angle, then the angles are supplementary. $\angle 1$ and $\angle 2$ are supplementary. So, you can conclude that $\angle 1$ and $\angle 2$ are adjacent.

Example 4 ***Using the Law of Syllogism***

Write some conditional statements that can be made from the following true statements using the Law of Syllogism.

1. If a cat is the largest of all cats, then it can weigh 650 pounds.
2. If a cat lives in a pride, then it is a lion.
3. If a cat weighs 650 pounds, then it is a tiger.
4. If a cat is a tiger, then it hunts alone.
5. If a cat is a lion, then it can weigh 400 pounds.

2.4 – Use Postulates and Diagrams

POINT, LINE, AND PLANE POSTULATES

Postulate 5 Through any two points there exists exactly one _____.

Postulate 6 A line contains at least two _______.

Postulate 7 If two lines intersect, then their intersection is _________________.

Postulate 8 Through any three ____________ points there exists exactly one plane.

Postulate 9 A plane contains at least three ____________ points.

Postulate 10 If two points lie in a plane, then the line containing them ________________.

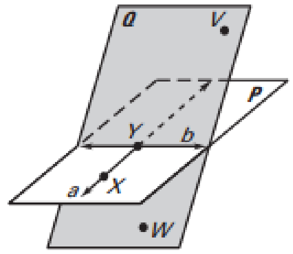
Postulate 11 If two planes intersect, then their intersection is a _____.

Example 1 ***Identify a postulate illustrated by a diagram***

State the postulate illustrated by the diagram.

Example 2 ***Identify postulates from a diagram***

Use the diagram to write examples of Postulates 9 and 11.



Postulate 9 Plane ____ contains at least three noncollinear points, ____________.

Postulate 11 The intersection of plane *P* and plane *Q* is ________.

Checkpoint **Use the diagram in Example 2 to complete the following exercises.**

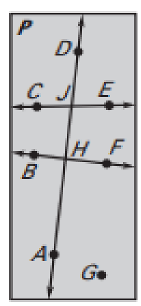
1. Which postulate allows you to say that the intersection of line *a* and line *b* is a point?

2. Write examples of Postulates 5 and 6.

CONCEPT SUMMARY: INTERPRETING A DIAGRAM

When you interpret a diagram, you can only assume information about size or measure if it is marked.

YOU CANNOT ASSUME



All points shown are ___________.

$\angle AHB$ and ________ are a linear pair.

$\angle AHF$ and ________ are vertical angles.

A, *H*, *J*, and *D* are ___________.

$\overleftrightarrow{AD}$ and $\overleftrightarrow{BF}$ intersect at ____.

YOU CANNOT ASSUME

G, *F*, and *E* are collinear.

$\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ intersect.

$\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ do not intersect.

$\angle BHA \cong \angle CJA$

$\overleftrightarrow{AD} \perp \overleftrightarrow{BF}$ or $m\angle AHB = 90°$

2.5 – Reason Using Properties from Algebra

ALGEBRAIC PROPERTIES OF EQUALITY

Let a, b, and c be real numbers.

Addition Property	If $a = b$, then ______________.
Subtraction Property	If $a = b$, then ______________.
Multiplication Property	If $a = b$, then _________.
Division Property	If $a = b$ and $c \neq 0$, then ______________.
Reflexive Property	For any real number a, ______.
Symmetric Property	If $a = b$, then ______.
Transitive Property	If $a = b$ and $b = c$, then ______.
Substitution Property	If $a = b$, then ______________________________.

Example 1 ***Writing Reasons***

Solve $-2x + 1 = 56 - 3x$ and write a reason for each step.

$-2x + 1 = 56 - 3x$	**Given**
$\_\_ + 1 = 56$	______________________
$x = \_\_$	______________________

Checkpoint **Solve the equation. Write a reason for each step.**

1. $12x - 3(x + 7) = 8x$

PROPERTIES OF EQUALITY

	Segment Length	Angle Measure
Reflexive	For any segment AB, ________.	For any angle A, ________.
Symmetric	If $AB = CD$, then ________.	If $m\angle A = m\angle B$, then ________.
Transitive	If $AB = CD$ and $CD = EF$, then ________.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then ________.

Example 3 ***Using Properties of Measure***

Use the information at the right to find $m\angle 1$.

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$$
$$m\angle 2 + m\angle 3 = m\angle 4$$
$$m\angle 1 = m\angle 4$$

Solution

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 =$ ________	**Given**
$m\angle 2 + m\angle 3 =$ ________	**Given**
$m\angle 1 =$ ________	**Given**
________ + ________ + ________ $= 360°$	**Substitution property of equality**
3(________) $= 360°$	**Simplify.**
________ = ________	**Division property of equality**
$m\angle 1 =$ ________	**Transitive property of equality**

Checkpoint **Complete the following exercise.**

2. In the diagram at the right, B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$. Show that $AB = CD$.

2.6 – Prove Statements about Segments and Angles

- **Proof –**

THEOREM 2.2 PROPERTIES OF ANGLE CONGRUENCE

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A, __________.

Symmetric If $\angle A \cong \angle B$, then __________.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then __________.

THEOREM 2.1 PROPERTIES OF SEGMENT CONGRUENCE

Reflexive For any segment AB, __________.

Symmetric If $\overline{AB} \cong \overline{CD}$, then __________.

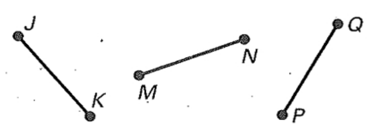
Transitive If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then __________.

Example 1 *Transitive Property of Segment Congruence*

You can prove the Transitive Property of Segment Congruence as follows.

Given: $\overline{JK} \cong \overline{MN}$, $\overline{MN} \cong \overline{PQ}$

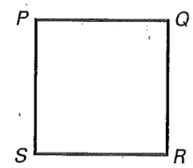
Prove: $\overline{JK} \cong \overline{PQ}$



Statements	Reasons
1. $\overline{JK} \cong \overline{MN}$; $\overline{MN} \cong \overline{PQ}$	1. __________
2. $JK = MN$, $MN = PQ$	2. __________
3. __________	3. Transitive property of equality
4. $\overline{JK} \cong \overline{PQ}$	4. Definition of congruent segments

Example 2 *Using Congruence*

Use the diagram and the given information to complete the proof.



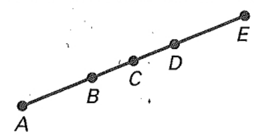
Given: $\overline{PQ} \cong \overline{RS}$, $\overline{PQ} \cong \overline{QR}$, $\overline{PS} \cong \overline{RS}$

Prove: $\overline{PS} \cong \overline{QR}$

Statements	Reasons
1. $\overline{PQ} \cong \overline{RS}$	1. Given
2. $\overline{PQ} \cong \overline{QR}$	2. ____________
3. $\overline{RS} \cong \overline{QR}$	3. Transitive Property of Congruence
4. $\overline{PS} \cong \overline{RS}$	4. ____________
5. $\overline{PS} \cong \overline{QR}$	5. Transitive Property of Congruence

Example 3 *Using Segment Relationships*

In the diagram, $AC = CE$ and $AB = DE$. Show that C is the midpoint of $\overline{BD}$.



Solution

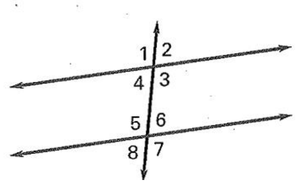
Given: ____________

Prove: ____________.

Statements	Reasons
1. $AC = CE$	1. ____________
2. $AB + BC = AC$	2. ____________
3. ____________	3. Transitive Property of Equality
4. $CD + DE = CE$	4. ____________
5. ____________	5. Transitive Property of Equality
6. $AB = DE$	6. ____________
7. $AB + BC = CD + AB$	7. ____________ ____________
8. ____________	8. Subtraction Property of Equality
9. ____________	9. Definition of congruent segments
10. C is the midpoint of $\overline{BD}$.	10. ____________

Example 1 *Using the Transitive Property*

In the diagram at the right, $\angle 1 \cong \angle 5$, $\angle 5 \cong \angle 3$, and $m\angle 1 = 103°$. What is the measure of $\angle 3$? Explain your reasoning.



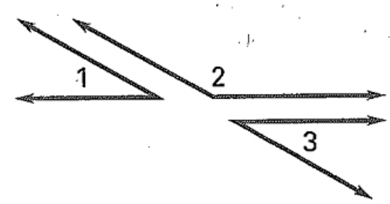
2.7 – Prove Angle Pair Relationships

THEOREM 2.3 RIGHT ANGLE CONGRUENCE THEOREM

All right angles are __________.

THEOREM 2.4 CONGRUENT SUPPLEMENTS THEOREM

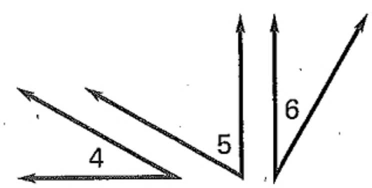
If two angles are supplementary to the same angle (or to congruent angles), then they are __________.



If $m\angle 1 + m\angle 2 =$ _____ and
$m\angle 2 + m\angle 3 =$ _____, then _________.

THEOREM 2.5 CONGRUENT COMPLEMENTS THEOREM

If two angles are complementary to the same angle (or to congruent angles), then the two angles are __________.

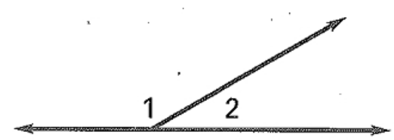


If $m\angle 4 + m\angle 5 =$ ____ and
$m\angle 5 + m\angle 6 =$ ____, then _________.

POSTULATE 12 LINEAR PAIR POSTULATE

If two angles form a linear pair, then they are _______________.

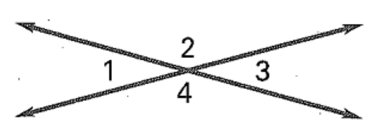
$m\angle 1 + m\angle 2 =$ _____



THEOREM 2.6 VERTICAL ANGLES THEOREM

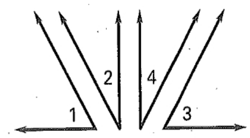
Vertical angles are __________.

$\angle 1 \cong$ ____ and ____ $\cong \angle 4$



Example 2 *Proving Theorem 2.5*

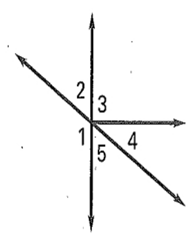
Given: ∠1 and ∠2 are complements,
∠3 and ∠4 are complements,
∠2 ≅ ∠4
Prove: ∠1 ≅ ∠3



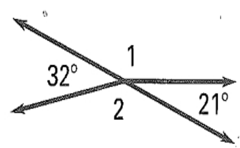
Statements	Reasons
1. ∠1 and ∠2 are complements, ∠3 and ∠4 are complements, ∠2 ≅ ∠4	**1.** ________
2. $m\angle 1 + m\angle 2 = 90°$, $m\angle 3 + m\angle 4 = 90°$	**2.** ________
3. ________	**3.** Transitive property of equality
4. $m\angle 2 = m\angle 4$	**4.** ________
5. ________	**5.** Substitution property of equality
6. ________	**6.** Subtraction property of equality
7. ∠1 ≅ ∠3	**7.** ________

Example 3 *Using Linear Pairs and Vertical Angles*

In the diagram, ∠3 is a right angle and $m\angle 5 = 57°$. Find the measures of ∠1, ∠2, ∠3, and ∠4.



2. Find $m\angle 1$ and $m\angle 2$.



3. Find the measure of each angle.