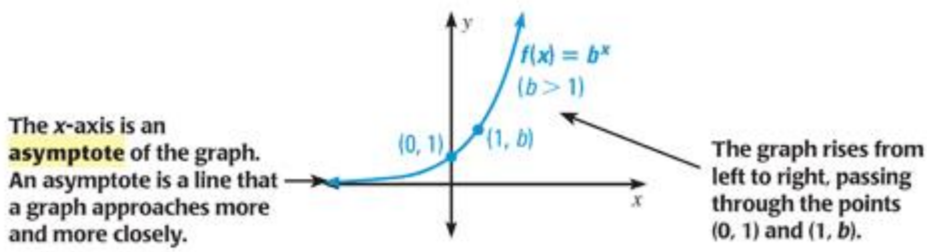
Algebra II – Chapter 7

7.1: Graph Exponential Growth Functions

- Have you heard of growing exponentially?
- An exponential function has the form $y = ab^x$, where b is a positive number (not 1)

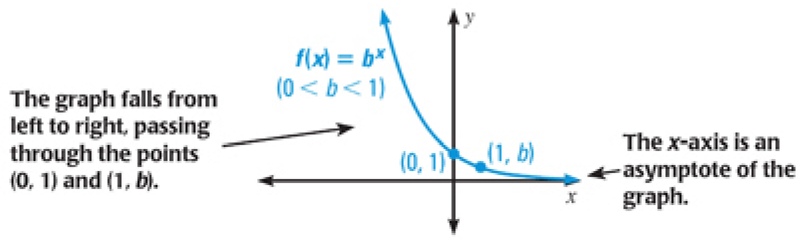
Exponential growth function:



Examples - Graph the function. State the domain and range.

1) $y = 4^x$
2) $y = \frac{1}{2} * 3^x$
3) $f(x) = 3^{x+1} + 2$

7.2: Graph Exponential Decay Functions



The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

Graph

1) $y = \left(\frac{2}{3}\right)^x$

2) $y = -2\left(\frac{3}{4}\right)^x$

Graph – state the domain and range

3) $y = \left(\frac{1}{4}\right)^{x-1} + 1$

4) $g(x) = -3\left(\frac{3}{4}\right)^{x-5} + 4$

7.3: Use Functions Involving e

In math, we have special numbers: π, i, e. Natural base, e, is known as the Euler number

Euler number is irrational. It's definition is: As n approaches $+\infty$, $\left(1+\frac{1}{n}\right)^n$ approaches $e \approx 2.71828182$

Simplify the expression:

1) $e^7 * e^4$

2) $2e^{-3} \cdot 6e^5$

3) $\frac{24e^8}{4e^5}$

4) $(10e^{-4x})^3$

5) Exponential growth or decay?

- $y = 1/2e^{4x}$
- $y = 2e^{-5x}$

Graph. State the domain and range.

5. $y = 2e^{0.5x}$

6. $y = 1.5e^{0.25(x-1)} - 2$

7.1 & 7.3 Interest

7.1 - Exponential growth model

Exponential Growth Models

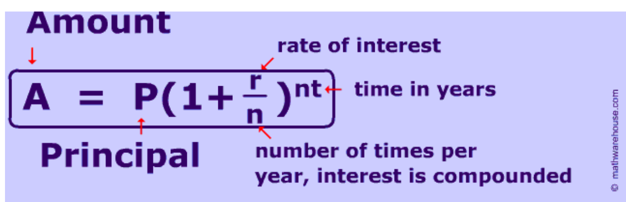
- When a real-life quantity increases by a fixed percent each year (or other period of time), the amount, y, after t years can be modeled by the equation:

$$y = a\underbrace{(1 + r)}_{\text{Growth Factor}}{}^{t}$$

a is the initial amount, r is the percent increase

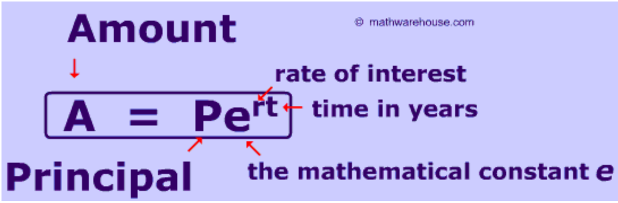
1) From 1997 to 2002, the number n (in millions) of DVD players sold in the US can be modeled by n=0.42(2.47)^t where t is the number of years since 1997. Identify the initial amount, the growth factor, and the annual percent increase. Graph and estimate the number of DVD players sold in 2001.

7.1 - Compound Interest is calculated using exponential growth functions.



1) You deposit $2000 in an account that pays 4% interest. Find the balance after 3 years if the interest is compounded daily.

7.3 - Continuously Compounded Interest



2) You deposit $2500 in an account that pays 5% annual interest compounded continuously. Find the balance after:

- 2 years

- 5 years

- 25 years

7.4: Evaluate Logarithms and Graph Logarithmic Functions

Logarithmic Functions
Any function of the form $f(x) = \log_b x$ where the logarithm of base b is defined as follows ($b > 0$ and $b \neq 1$)
$y = \log_b x \Leftrightarrow b^y = x$

The logarithm goes to the basement to find the answer which equals the exponent.

Rewrite in exponential form or evaluate without using a calculator:

1) $\log_3 81 = 4$
2) $\log_7 7 = 1$
3) $\log_5 0.2$
4) $\mathbf{\log_{36} 6}$
5) $\boldsymbol{Use\ a\ calculator\ to\ evaluate: log\ 0.746}$

Common Logarithm
Using **10 as the base**
$y = \log_{10} x = \log x$

Natural Logarithm
Using ***e*** **as the base**
$y = \log_e x = \ln x$

Inverse Property of Logs
$\log_b b^x = x$
$b^{\log_b x} = x$

6) Simplify: $7^\wedge(\log_7 x)$
7) Simplify: $\log_5 125^x$
8) Find the inverse of the function: $y = 2^x - 3$
9) Graph: $y = \log_6 x$

7.5: Apply Properties of Logarithms

Property	Definition	Example
Product	$\log_b mn = \log_b m + \log_b n$	$\log_3 9x = \log_3 9 + \log_3 x$
Quotient	$\log_b \frac{m}{n} = \log_b m - \log_b n$	$\log_{\frac{1}{4}} \frac{4}{5} = \log_{\frac{1}{4}} 4 - \log_{\frac{1}{4}} 5$
Power	$\log_b m^p = p \cdot \log_b m$	$\log_2 8^x = x \cdot \log_2 8$
Equality	If $\log_b m = \log_b n,$, then $m = n$.	$\log_8(3x-4) = \log_8(5x+2)$ so, 3x – 4 = 5x+2

Use $\log 5 \approx 0.898$ and $\log 8 \approx 1.161$ to evaluate:

1) $log(\frac{5}{8})$

2) $\log 40$

3) Expand the expression: $3x^4$

4) Condense the expression: $\ln 4 + 3\ln 3 - \ln 12$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

5) Evaluate using change of base: $\log_8 14$

7.6: Solve Exponential and Logarithmic Equations

If $b^x = b^y$ then x=y

1) Solve: $9^{2x} = 27^{x-1}$

2) Solve: $100^{7x+1} = 1000^{3x-2}$

3) Solve: $81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$

What if we can't rearrange? Then we use log or ln.

4) Solve: $2^x = 5$

5) Solve: $7^{9x} = 15$

6) Solve: $4e^{-0.3x} - 7 = 13$

If log x = log y, then x=y

7) $(5x + 9) \quad = 6x$

8) $ln \ (x + 19) = ln \ (7x + 8)$

9) $\log_4(-x) + \log_4(x + 10) = 2$

7.7: Write and Apply Exponential and Power Functions

Write an exponential function $\boldsymbol{y = ab^x}$ whose graph passes through the given points:

$\mathbf{(1, 40), (3, 640)}$

Use the points (x, y) to find a model for the data:

$\mathbf{(1, 3.3), (2, 10.1), (3, 30.6), (4, 92.7), (5, 280.9)}$

Write a power function $\boldsymbol{y = ax^b}$ whose graph passes through the given points:

$\mathbf{(3, 14), (9, 44)}$