Algebra II – Chapter 6

6.1: Evaluate nth Roots and Use Rational Exponents

$$\sqrt{a} = a^{1/2}$$

$$\sqrt[n]{a} = a^{1/n}$$

n is an even integer	n is an odd integer
a < 0 No real nth roots	**a < 0** One real nth root
a = 0 One real nth root	**a = 0** One real nth root
a > 0 Two real nth roots	**a > 0** One real nth root

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

$$a^{\frac{-m}{n}} = \frac{1}{\left(\sqrt[n]{a}\right)^m}$$

Examples:

1) $n = 3, a = -216$

2) $n = 4, a = 81$

3) Simplify: $25^{3/2}$

4) Simplify: $\left(\sqrt[3]{8}\right)^{-2}$

5) Simplify: $\sqrt[3]{-125}$

6) Solve: $x^3 = 64$

7) Solve: $\frac{1}{2}x^5 = 512$

8) $(x-2)^3 = -14$

6.2: Properties of Rational Exponents

Product of Powers	$a^m \cdot a^n = a^{m+n}$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^m = a^m b^m$
Negative Exponents	$a^{-m} = \frac{1}{a^m}, a \neq 0$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

1) $\left(6^{2/3}\right)^{1/2}$

2) $\frac{9}{9^{-4/5}}$

3) $120^{-2/5} \cdot \frac{120^{2/5}}{7^{-3/4}}$

4) $\frac{13^{3/7}}{13^{5/7}}$

5) $\sqrt[3]{108} \cdot \sqrt[3]{4}$

6) $\sqrt[3]{\frac{1}{6}}$

7) $\sqrt[6]{\frac{81}{4}}$

8) $2\sqrt[4]{3} + 7\sqrt[4]{3}$

9) $\frac{x^{\left(\frac{2}{5}\right)}y}{xy^{-\frac{1}{3}}}$

6.3: Perform Function Operations and Composition

If f and g are two functions, a new function can be created h	
Adding	$h(x) = f(x) + g(x)$
Subtracting	$h(x) = f(x) - g(x)$
Multiplying	$h(x) = f(x)g(x)$
Dividing	$h(x) = \frac{f(x)}{g(x)}$

1) $f(x) = -\boldsymbol{3}x^{\boldsymbol{1/3}} + \boldsymbol{4}x^{\boldsymbol{1/2}}$ and $g(x) = \boldsymbol{5}x^{\boldsymbol{1/3}} + \boldsymbol{4}x^{\boldsymbol{1/2}}$
   a. $g(x) + f(x)$
   b. $g(x) - f(x)$
2) $h(x) = \boldsymbol{4}x^{\boldsymbol{2/3}}$ and $t(x) = \boldsymbol{5}x^{\boldsymbol{1/2}}$
   a. $h(x) \cdot t(x)$
   b. $\frac{h(x)}{t(x)}$

Compositions of functions. Indicate the domain.

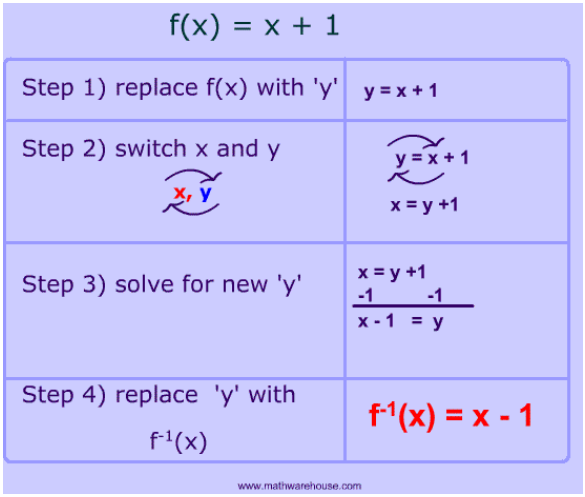
3) $f(x) = \boldsymbol{3}x - \boldsymbol{8}$ and $g(x) = \boldsymbol{2}x^{\boldsymbol{2}}$
   a. $f\big(g(x)\big)$
   b. $g\big(f(x)\big)$
   c. $g\big(f(x)\big)$
   d. $f\big(g(\boldsymbol{5})\big)$

6.4 – Use Inverse Functions

Inverse relation -> Switches the input and output values. Also, the domain and range are interchanged.

An inverse function if written as $f^{-1}(x)$

Two functions are inverses if: $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$



1) Find the inverse: $f(x) = \mathbf{2}x - \mathbf{1}$

2) Find the inverse: $f(x) = -\mathbf{2}/\mathbf{3}x + \mathbf{2}$

3) Verify inverse functions: $f(x) = \mathbf{2}x + \mathbf{3}$ and $g(x) = \frac{\mathbf{1}}{\mathbf{2}}x - \frac{\mathbf{3}}{\mathbf{2}}$

4) Verify inverse functions: $f(x) = \frac{\mathbf{1}}{\mathbf{5}}x - \mathbf{1}$ and $g(x) = \mathbf{5}x + \mathbf{5}$

5) Find the inverse of the power function: $f(x) = -\mathbf{10}x^{\mathbf{6}} \;\; x <= \mathbf{0}$

6.5 – Graph Square Root and Cube Root Functions

Graph and state the domain and range:

1) $y = -3\sqrt{x}$

2) $y = \frac{1}{2}\sqrt[3]{x}$

3) $y = -2\sqrt[3]{x+5} + 3$

6.6 – Solve Radical Equations

1) $\sqrt[3]{x} - 9 = -1$

2) $\sqrt{x + 25} = 4$

3) $2\sqrt[3]{x - 3} = 4$

4) $3x^{3/2} = 375$

5) $(x + 2)^{1/3} + 3 = 7$

6) $\sqrt{x + 3} = 3 - \sqrt{x}$