Algebra II – Chapter 5

5.1: Use Properties of Exponents

Property	Example
$(xy)^m = x^m \cdot y^m$	$(xy)^3 = x^3y^3$
$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	$\left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$
$x^m \cdot x^n = x^{m+n}$	$x^4 \cdot x^2 = x^6$
$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^5}{x^3} = x^2$
$(x^m)^n = x^{mn}$	$(x^4)^2 = x^8$
$x^1 = x$	$473{,}837{,}843^1 = 473{,}837{,}843$
$x^0 = 1$	$473{,}837{,}843^0 = 1$
$\frac{1}{x^m} = x^{-m}$	$\frac{1}{3^2} = 3^{-2} = \frac{1}{8}$

1) $(4^{-2})^3$

2) $\left(\frac{2}{3}\right)^{-5}\left(\frac{2}{3}\right)^4$

3) $(5s^{-2}t^4)^{-3}$

4) $\frac{4r^4s^5}{24r^4s^{-5}}$

5) $\frac{y^{11}}{4z^3} * \frac{8z^7}{y^7}$

6) $(1.2 * 10^{-3})(6.7 * 10^{-7})$

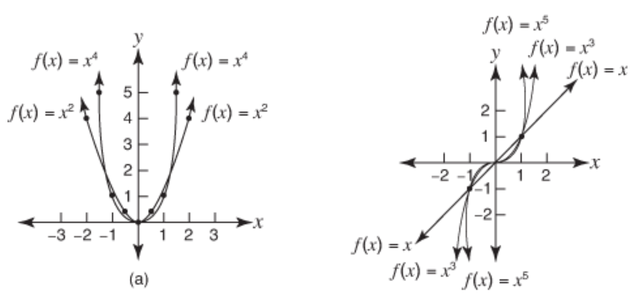
7) $\frac{1.1*10^{-3}}{5.5*10^{-8}}$

5. 2 Evaluate and Graph Polynomial Functions

Degree	Type	Standard Form
0	Constant	$f(x) = a_0$
1	Linear	$f(x) = a_1x + a_0$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$
4	Quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

$f(x) = x^n$, n an even integer

$f(x) = x^n$, n an odd integer



(a)

(b)

Is it a function? If it is a function, write in standard form and state degree, type, and leading coefficient. Also graph it.

1) $\boldsymbol{f(x) - 6x + 8x^4 - 3}$

2) $\boldsymbol{h(x) = x^3\sqrt{10} + 5x^{-2} + 1}$

3) $\boldsymbol{g(x) = 8x^3 - 4x^2 + \frac{2}{x}}$

Use direct substitution to evaluate the polynomial:

4) $\boldsymbol{f(x) = 8x + 5x^4 - 3x^2 - x^3; x = 2}$

Use synthetic substitution to evaluate the polynomial:

5) $\boldsymbol{g(x) = 6x^5 + 10x^3 - 27; x = -3}$

5.3: Add, Subtract, and Multiply Polynomials

Find the sum or difference:

1) $(t^2 - 6t + 2) + (5t^2 - t - 8)$

2) $(8v^4 - 2v^2 + v - 4) - (3v^3 - 12v^2 + 8v)$

Multiply the polynomial:

3) $x(2x^2 - 5x + 7)$

4) $(y - 7)(y + 6)$

5) $(w + 4)(w^2 + 6w - 11)$

6) $(5c^2 - 4)(2c^2 + c - 3)$

7) $(z - 4)(-z + 2)(z + 8)$

5.4: Factor and Solve Polynomial Equations

KEY CONCEPT *For Your Notebook*

Special Factoring Patterns

Sum of Two Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Example

$8x^3 + 27 = (2x)^3 + 3^3$

$= (2x + 3)(4x^2 - 6x + 9)$

Difference of Two Cubes

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example

$64x^3 - 1 = (4x)^3 - 1^3$

$= (4x - 1)(16x^2 + 4x + 1)$

Factor the polynomial completely:

1) $x^3 - 6x^2 - 72x$

2) $27m^3 + 1$

3) $y^3 - 8$

4) $n^3 + 5n^2 - 9n - 45$

5) $36m^6 + 12m^4 + m^2$

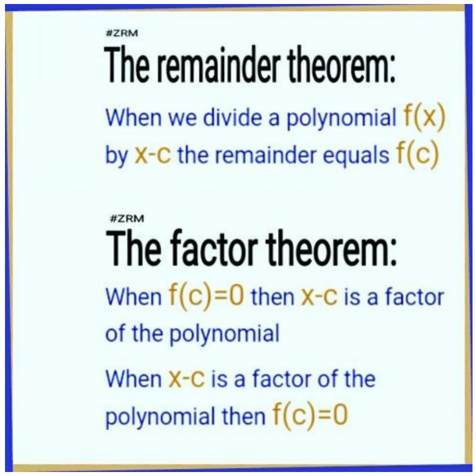
6) $3s^4 - s^2 - 24$

Solve by factoring:

7) $y^3 - 5y^2 = 0$

8) $5b^3 + 15b^2 + 12b = -36$ (real solutions only)

5.5: Apply the Remainder and Factor Theorems



Divide the polynomial using long division and synthetic division:

1) $(x^3 - x^2 + 4x - 10) \div (x + 2)$

2) $(4x^3 + x^2 - 3x + 7) \div (x - 1)$

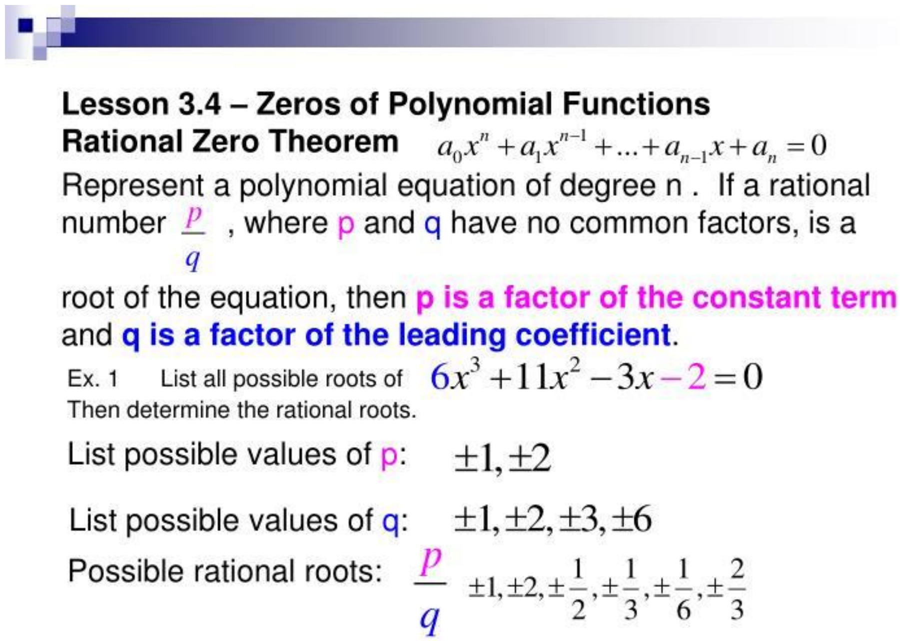
Factor the polynomial completely given that x-4 is a factor.

3) $f(x) = x^3 - x^2 - 22x + 40$

Given polynomial function f and a zero of f, find the other zeros.

4) $f(x) = 3x^3 + 34x^2 + 72x - 64;\ -4$

5.6: Find Rational Zeros



List the possible rational zeros of f using the rational zero theorem:

1) $f(x) = 2x^3 + x^2 - x - 18$

Find all the real zeros of the function:

2) $f(x) = x^3 - 8x^2 + 5x + 14$

3) $f(x) = 3x^3 + 19x^2 + 4x - 12$

5.7: Apply the Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra:
Every polynomial with a degree greater than 0 has at least one root in the set of complex numbers
Corollary to the Fundamental Theorem of Algebra:
Every polynomial of degree *n* can be factored into *n* linear factors multiplied by some constant ($k \neq 0$): $k(x - r_1)(x - r_2)(x - r_3)...(x - r_n)$

How many solutions does the equation have:

1) $x^4 + 5x^2 - 36 = 0$

2) $f(x) = x^3 + 7x^2 + 8x - 16$

Find all the zeros of the polynomial functions:

3) $f(x) = x^3 + 7x^2 + 15x + 9$

4) $f(x) = x^5 - 2x^4 + 8x^2 - 13x + 6$

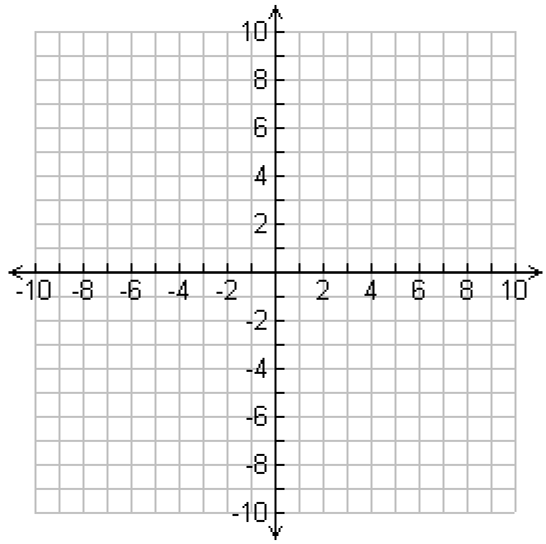
Write a polynomial function f of at least the degree that had rational coefficients, a leading coefficient of 1, and the given zeros:

5) $-1, 2, 4$

6) $2, i, -i$

5.8: Analyze Graphs of Polynomial Functions

Graph the function: $h(x) = \frac{1}{12}(x-1)(x+4)(x+8)$



ANALYZING GRAPHS **Estimate the coordinates of each turning point and state whether each corresponds to a local maximum or a local minimum. Then estimate all real zeros and determine the least degree the function can have.**

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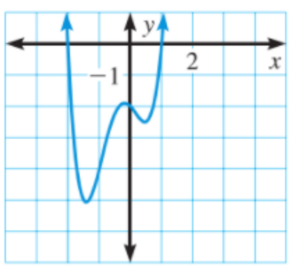

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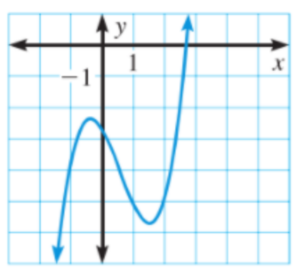

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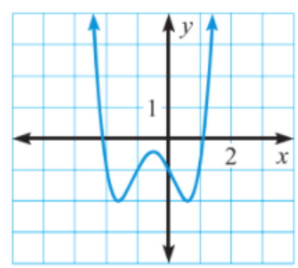


5.9: Write Polynomial Functions and Models

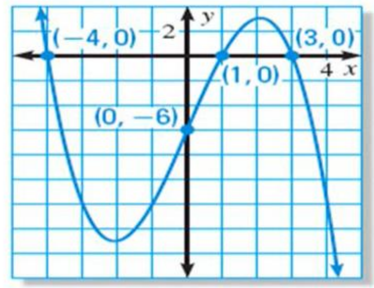
Write the cubic function whose graph is shown.

SOLUTION

STEP 1

Use the three given x - intercepts to write the function in factored form.

$f(x) = a\,(x + 4)(x - 1)(x - 3)$



STEP 2

Find the value of a by substituting the coordinates of the fourth point.

Write a cubic function whose graph passes through the points.

$(-3, 0), (1, 0), (3, 2), (4, 0)$