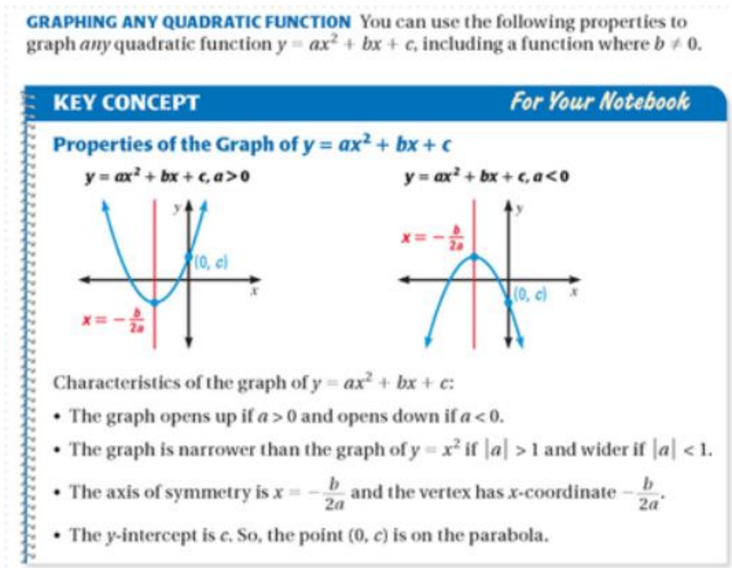
Algebra II – Chapter 4

4.1: Graph Quadratic Functions in Standard Form

Now we are going to switch from linear equations to quadratic equations. Quadratic equations are in the form: $\boldsymbol{x^2 + bx + c = y}$.



GRAPHING ANY QUADRATIC FUNCTION You can use the following properties to graph *any* quadratic function $y = ax^2 + bx + c$, including a function where $b \neq 0$.

KEY CONCEPT *For Your Notebook*

Properties of the Graph of $y = ax^2 + bx + c$

Characteristics of the graph of $y = ax^2 + bx + c$:

- The graph opens up if $a > 0$ and opens down if $a < 0$.
- The graph is narrower than the graph of $y = x^2$ if $|a| > 1$ and wider if $|a| < 1$.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex has x-coordinate $-\frac{b}{2a}$.
- The y-intercept is c. So, the point $(0, c)$ is on the parabola.

Graph the function. Compare to $y = x^2$. Label the vertex and axis of symmetry. Tell whether the function has a minimum of maximum value and state the value.

1) $y = 5x^2$

2) $y = 4x^2 + 1$

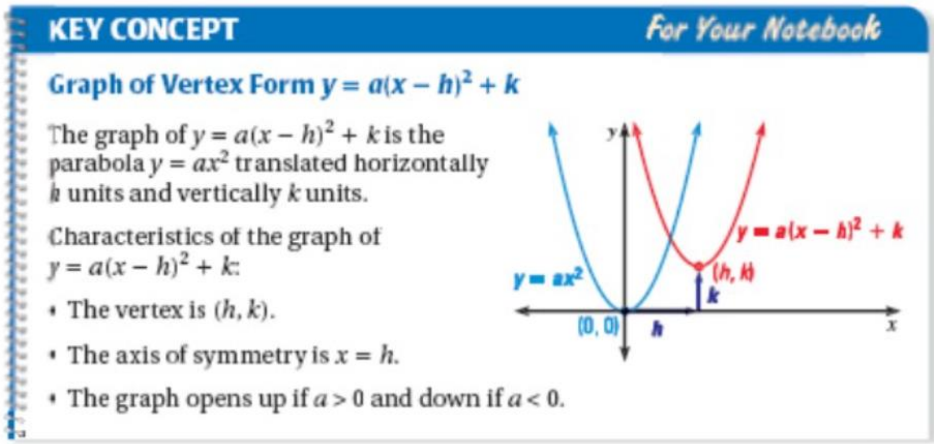
3) $y = 3x^2 - 6x + 4$

4) $f(x) = \frac{1}{2}x^2 + x - 3$

4.2: Graph Quadratic in Vertex or Intercept Form

Standard Form: $y = ax^2 + bx + c$

Vertex Form: $y = a(x - h)^2 + k$

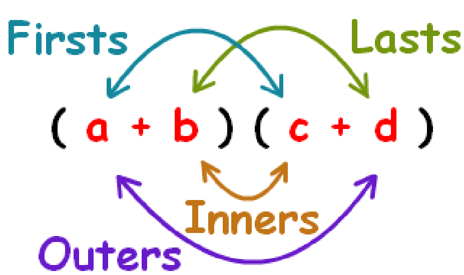


1. **Graph the function and label the vertex and axis of symmetry:**

   $y = (x + 4)^2 + 2$

   $y = -\frac{1}{4}(x + 2)^2 + 1$

Remember FOIL and Factoring (unFOIL) ?



2. **Write the quadratic function in standard form.**

   $f(x) = 4(x + 1)(x - 6)$

   $y = -(x + 6)^2 + 10$

4.3: Solve $x^2 + bx + c = 0$ by Factoring

- **How can we factor: $x^2 - 3x - 18$?**

Some special cases:

Difference of squares: $\boldsymbol{a^2 - b^2 = (a+b)(a-b)}$

Perfect squares: $\boldsymbol{a^2 + 2ab + b^2 = (a+b)^2}$

Perfect squares: $\boldsymbol{a^2 - 2ab + b^2 = (a-b)^2}$

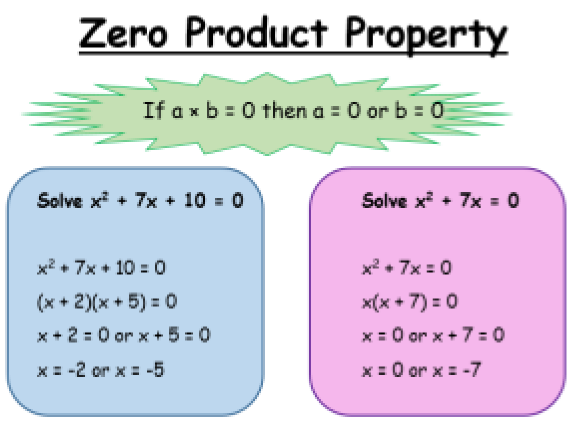
Factor or tell if unfactorable:

1) $\boldsymbol{x^2 - 7x + 10}$

2) $\boldsymbol{n^2 - 3n + 9}$

3) $\boldsymbol{q^2 - 100}$

4) $\boldsymbol{w^2 - 18w + 81}$



Solve the equation:

5) $\boldsymbol{x^2 + 2x - 35 = 0}$

6) $\boldsymbol{r^2 + 2r = 80}$

4.4: Solve $ax^2 + bx + c = 0$ by Factoring

What happens if we have a number in front of the x^2term? We still factor!

Practice:

1) $3n^2 + 7n + 4$

2) $4r^2 - 25$

3) $25t^2 - 30t + 9$

One way to help is to factor the monomial 1st…

4) $12x^2 - 4x - 40$

5) $-8y^2 + 28y - 60$

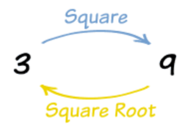
We can also solve if we remember that A*B=0, either A=0 or B=0

6) Solve $14s^2 - 21s = 0$

7) Solve: $4s^2 - 20x + 25 = 0$

4.5: Solve Quadratic Equations Finding Square Roots

Do you remember square roots? Another way to solve quadratic equations is to find the square roots.



There are a few rules for square roots:

- **Product Property:** $\sqrt{\boldsymbol{a \cdot b}} = \sqrt{\boldsymbol{a}} \cdot \sqrt{\boldsymbol{b}}$

- **Quotient Property:** $\sqrt{\frac{\boldsymbol{a}}{\boldsymbol{b}}} = \frac{\sqrt{\boldsymbol{a}}}{\sqrt{\boldsymbol{b}}}$

<u>Practice</u>:

1) $\sqrt{\mathbf{27}}$

2) $\sqrt{\mathbf{10}} \cdot \sqrt{\mathbf{15}}$

3) $\sqrt{\frac{\mathbf{15}}{\mathbf{4}}}$

4) $\frac{\mathbf{1}}{\mathbf{5}+\sqrt{\mathbf{6}}}$

Solve

5) $\mathbf{5}\boldsymbol{x}^{\mathbf{2}} = \mathbf{80}$

6) $\boldsymbol{z}^{\mathbf{2}} - \mathbf{7} = \mathbf{29}$

7) $\mathbf{3}(\boldsymbol{x} - \mathbf{2})^{\mathbf{2}} = \mathbf{40}$

4.6: Perform Operations with Complex Numbers

What happens when you want to take the square root of a negative number? Imaginary numbers!

$$\boldsymbol{i = \sqrt{-1}\ \ which\ means\ i^2 = -1}$$

So, $\boldsymbol{\sqrt{-3} = i\sqrt{3}}$

You can also have a mixture of real and imaginary numbers, called a complex number, like $\boldsymbol{3 + 8i}$

Practice:

1) Solve: $\boldsymbol{x^2 + 11 = 3}$

2) Solve: $\boldsymbol{5x^2 + 33 = 3}$

When adding/subtracting/multiplying/dividing imaginary numbers, treat the $\boldsymbol{i}$ like a variable, but remember $\boldsymbol{i^2 = -1}$

3) $\boldsymbol{(9 - i) + (-6 + 7i)}$

4) $\boldsymbol{(3 + 7i) - (8 - 2i)}$

5) $\boldsymbol{-4 - (1 + i) - (5 + 9i)}$

6) $\boldsymbol{6i(3 + 2i)}$

7) $\boldsymbol{(-1 - 5i)(-1 + 5i)}$

4.7: Complete the Square

Another method to solve quadratics is to complete the square.

To complete the square for $\boldsymbol{x^2 + bx}$ add $\left(\frac{b}{2}\right)^2$

<u>Practice</u>:

Solve by square roots:

1) $\boldsymbol{x^2 + 6x + 9 = 36}$

2) $\boldsymbol{x^2 - 10x + 25 = 1}$

3) $\boldsymbol{x^2 - 24x + 144 = 100}$

Complete the square:

4) $\boldsymbol{x^2 + 22x + c}$

Solve the equation by completing the square:

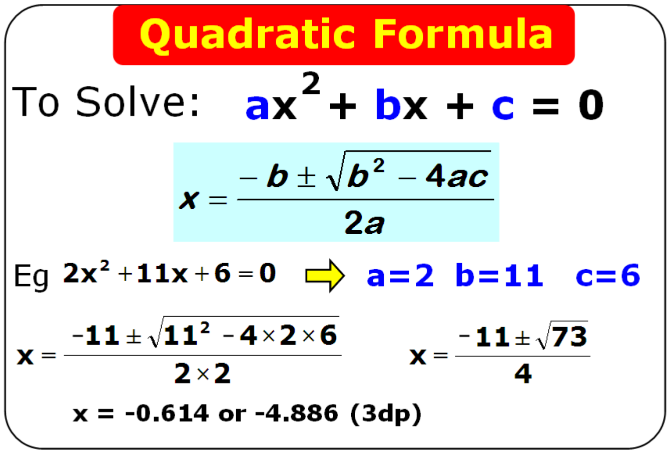
5) $\boldsymbol{x^2 + 6x + 4 = 0}$

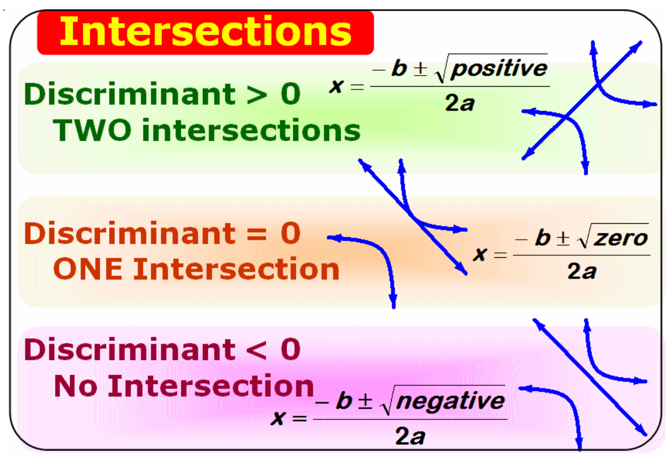
6) $\boldsymbol{3x^2 + 12x - 18 = 0}$

7) $\boldsymbol{4p(p - 2) = 100}$

4.8: Using the Quadratic Formula and the Discriminant

For any quadratic, you can use the quadratic formula to solve:





Practice: Use the quadratic formula to solve the equation:

1) $x^2 - 6x + 7 = 0$

2) $7x - 5 + 12x^2 = -3x$

3) Check using factoring: $z^2 + 15z + 24 = -32$

4) Find the discriminate and tell the number of solutions: $5x^2 + 16x = 11x - 3x^2$

4.9: Graph and Solve Quadratic Inequalities

We can also graph the quadratic inequalities.

1) $y < -x^2$

2) $y > -2x^2 + 9x - 4$

3) $y \geq 2x^2$ and $y < -x^2 + 1$

4) $y > 3x^2 + 3x - 5$ and $y < -x^2 + 5x + 10$

5) Solve by graphing: $x^2 + 8x \leq -7$

6) Solve by graphing: $-\frac{1}{2}x^2 + 4x \geq 1$

4.10: Write Quadratic Functions and Models

Write a quadratic function in vertex form:

1. Vertex (5, -4) Point (1, 20)
2. Vertex (-1, -4) Point (2, -1)

Write a quadratic function in standard form:

3. $(-2, -4), (0, -10), (3, -7)$