Algebra II – Chapter 3

3.1: Solve Linear Systems by Graphing

What is a system? A system is two equations that can be written in the following way:

$$\boldsymbol{Ax + By = C}$$

$$\boldsymbol{Dx + Ey = F}$$

We can find a solution to a system of equations by finding the values of $\boldsymbol{x}$ and $\boldsymbol{y}$ that make both true.

In this section, we are going to solve using the graphing method. We will graph both lines and see where they intersect. Where they intercept is the solution.

<u>Practice</u>:

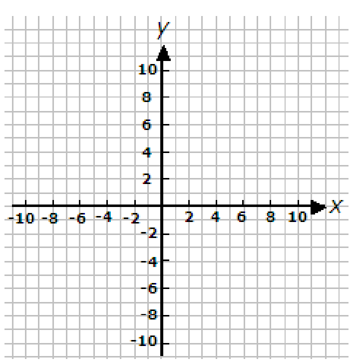
Graph the linear system and check algebraically.

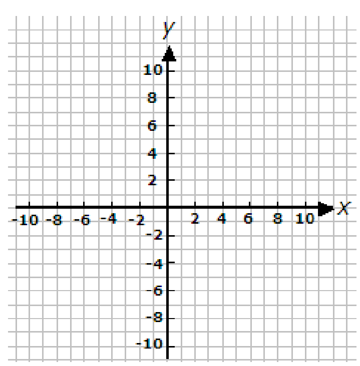
1) $x + 2y = 2$
   $x - 4y = 14$

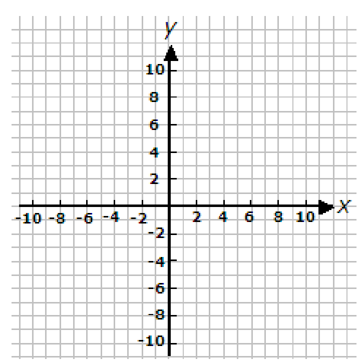
2) $2x - y = 4$
   $x - 2y = -1$

3) $y = 4x + 3$
   $20x - 5y = -15$

4) $y = 2x - 1$
   $-6x + 3y = -12$







3.2: Solve Linear Systems Algebraically

Solve using the <u>substitution</u> method:

$$4x + 3y = -2$$

$$x + 5y = -9$$

1) Solve one equation for a variable.

   $x = -9 - 5y$

2) Substitute the equation into the other equation.
   $4x + 3y = -2$
   $4(-9 - 5y) + 3y = -2$
   $-36 - 20y + 3y = -2$
   $-36 - 17y = -2$
   $-17y = 34$
   $y = -2$
3) Substitue the solution back into step 1.
   $x = -9 - 5(-2)$
   $x = 1$

Solve using the <u>elimination</u> method:

$$5x + 5y = 5$$

$$5x + 3y = 4.2$$

1) Get one variable in common with both equations

   $5x$

2) Add or subtract the equations to eliminate a variable.
   $5x + 5y = 5$
   $\underline{-(5x + 3y = 4.2)}$
   $2y = 0.8$
   $y = 0.4$
3) Substitue the solution into one of the equations to solve.
   $5x + 5y = 5$
   $5x + 5(0.4) = 5$
   $x = 0.6$

Practice:

1) $5x + 3y = 20$
$-x - 3y = -4$

2) $8x + 9y = 15$
$5x - 2y = 17$

3) $12x - 3y = -9$
$-4x + y = 3$

4) $6x + 15y = -12$
$-2x - 5y = 9$

3.3: Solve Linear Systems Algebraically

Practice – graph the system of inequalities.

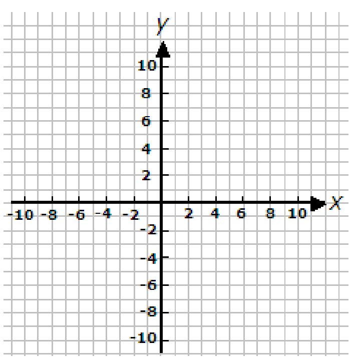
1) $y \leq 3x - 2$
   $y > -x + 4$

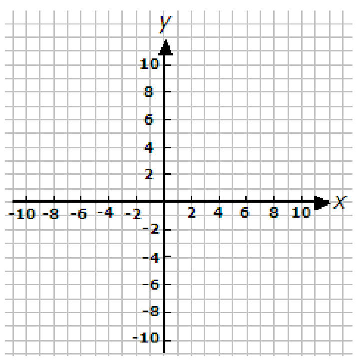
2) $x + y > -3$
   $-6x + y < 1$

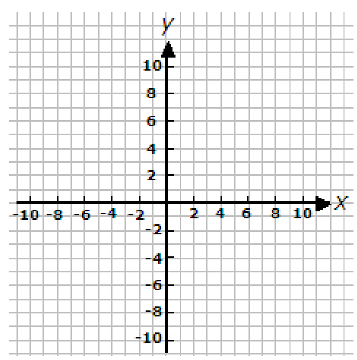
3) $y > -2$
   $y \leq -|x + 2|$

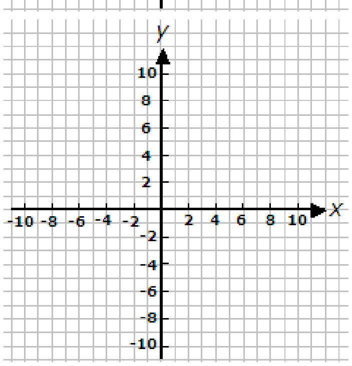
4) $x \geq 8$
   $y \leq -1$
   $y < -2x - 4$

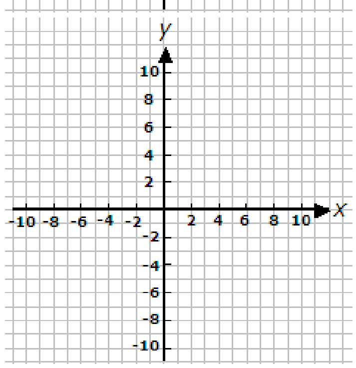
5) $x + y < 5$
   $x + y > -5$
   $x - y < 4$
   $x - y > -2$

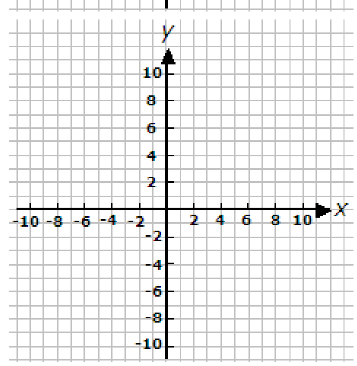












3.4: Solve Systems of Linear Equations in Three Variables

We can now solve equations that have three variables!

The equations is in the form of: $ax + by + cz = d$ and the solution is (x, y, z).

We use the same strategy, either elimination or substitution method, or a combination of both.

Practice:

1. Solve:
   $3x + y - 2z = 10$
   $6x - 2y + z = -2$
   $x + 4y + 3z = 7$

2. $x + y - z = 2$
   $2x + 2y - 2z = 6$
   $5x + y - 3z = 8$

3.5: Perform Basic Matrix Operations

(http://www.mathsisfun.com/algebra) What is a matrix? A Matrix is an array of numbers:

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

A Matrix

(This one has 2 Rows and 3 Columns)

Adding/Subtracting

To add two matrices: add the numbers in the matching positions. The two matrices **must** be the same size to add or subtract!

3+4=7

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

These are the calculations:

3+4=7	8+0=8
4+1=5	6-9=-3

Scalar Multiplication:

2x4=8

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

These are the calculations:

2×4=8	2×0=0
2×1=2	2×-9=-18

We call the constant a **scalar**, so officially this is called "scalar multiplication".

Practice:

1. $\begin{bmatrix} 5 & 2 \\ -1 & 8 \\ 4 & -5 \end{bmatrix} + \begin{bmatrix} -8 & 10 \\ -6 & 3 \\ 2 & -1 \end{bmatrix}$

2. $\begin{bmatrix} 1.2 & 5.3 \\ 6.1 & 4.4 \end{bmatrix} - \begin{bmatrix} 2.4 & -0.6 \\ 0.1 & 3.1 \end{bmatrix}$

3. $2\begin{bmatrix} -1 & 4 \\ 3 & -6 \end{bmatrix}$

4. Solve the matrix equation:

$$2\begin{bmatrix} 8 & -x \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 3 & -9 \\ 10 & -4y \end{bmatrix} = \begin{bmatrix} 13 & 4 \\ 0 & 16 \end{bmatrix}$$

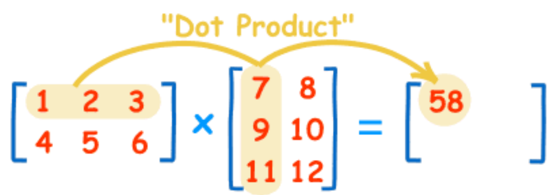
3.6: Multiply Matrices

To multiply an m×n matrix by an n×p matrix, the ns must be the same,

and the result is an m×p matrix.

But to multiply a matrix **by another matrix** we need to do the " dot product " of rows and columns:

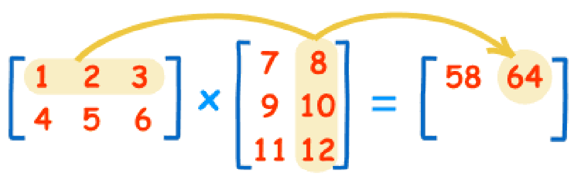
To work out the answer for the **1st row** and **1st column**:



The "Dot Product" is where we **multiply matching members**, then sum up:

(1, 2, 3) • (7, 9, 11) = 1×7 + 2×9 + 3×11 = 58

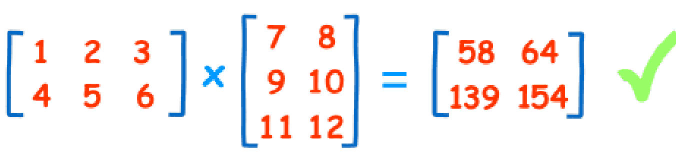
Want to see another example? Here it is for the 1st row and **2nd column**:



(1, 2, 3) • (8, 10, 12) = 1×8 + 2×10 + 3×12 = 64

We can do the same thing for the **2nd row** & **1st column**:(4, 5, 6) • (7, 9, 11) = 4×7 + 5×9 + 6×11 = 139

And for the **2nd row** and **2nd column**: (4, 5, 6) • (8, 10, 12) = 4×8 + 5×10 + 6×12 = 154



3.7: Evaluate Determinates and Apply Cramer's Rule

3.8: Use Inverse Matrices to Solve Linear Systems,

$$\det(\mathbf{A}) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\det(\mathbf{A}) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

We use the determinate to find the inverse, which helps us solve problems:

Well, for a 2x2 Matrix the Inverse is:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\underbrace{ad-bc}_{\text{determinant}}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Say that you know Matrix A and B, and want to find Matrix X: $XA = B$

It would be nice to divide both sides by A (to get X=B/A), but remember **we can't divide**. But what if we multiply both sides by A^{-1} ? $XAA^{-1} = BA^{-1}$

And we know that $AA^{-1} = I$, so: $XI = BA^{-1}$

We can remove I (for the same reason we could remove "1" from 1x = ab for numbers): $X = BA^{-1}$

And we have our answer (assuming we can calculate A^{-1})

3.6, 3.7, 3.8 Practice:

3.6 – 14, 22, 24

3.7 – 4, 12

3.8 – 6, 26, 32

3.8 - #26

$$4x + 7y = -16$$

$$2x + 3y = -4$$

Can be rewritten as:

$$\begin{bmatrix} 4 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -16 \\ -4 \end{bmatrix} \quad or\ [A]X = [B]$$

The inverse of

$\begin{bmatrix} 4 & 7 \\ 2 & 3 \end{bmatrix}$ is $\begin{bmatrix} -1.5 & 3.5 \\ 1 & -2 \end{bmatrix}$ or $[A]^{-1}$

$\begin{bmatrix} 4 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -1.5 & 3.5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1.5 & 3.5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -16 \\ -4 \end{bmatrix}$ or $[A][A]^{-1}X = [A]^{-1}[B]$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1.5 & 3.5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -16 \\ -4 \end{bmatrix}$ or $X = [A]^{-1}[B]$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$$