Algebra II: Chapter 13

13.1 – Use Trigonometry with Right Triangles

VOCABULARY

Right Triangle Definition of Trigonometric Functions

Let θ be an acute angle of a right triangle. The six trigonometric functions of θ are defined as follows.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

The abbreviations *opp*, *adj*, and *hyp* represent the lengths of the three sides of the right triangle. Note that the ratios in the second row are the reciprocals of the ratios in the first row. That is:

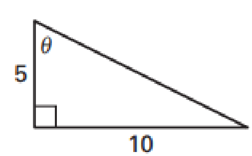
$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

The table below gives values of the six trigonometric functions for the common angles 30°, 45°, and 60°.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

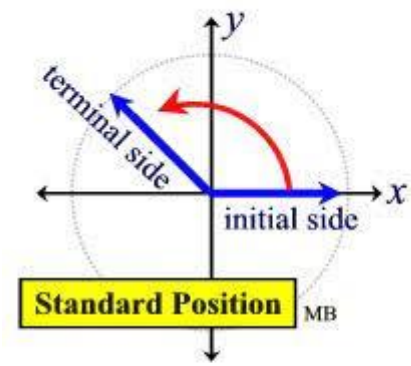
Finding all missing side lengths and angle measures is called **solving a right triangle**.

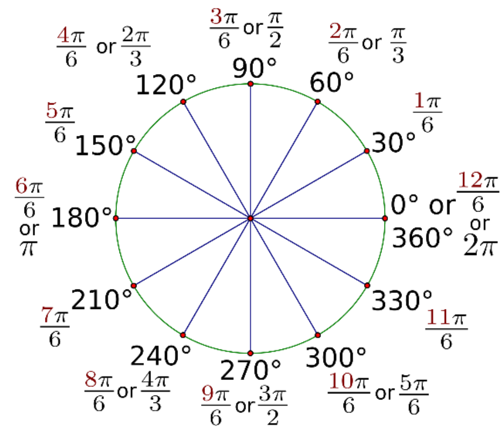
Evaluate the six trigonometric functions of the given angle θ.



- Solve with the given information $B = 53°, a = 12$

13.2 – Define General Angles and Use Radian Measures





Draw an angle with the given measure in standard position. Then tell in which quadrant the terminal side lies.

a. 120° **b.** −210° **c.** 570°

- **Find one positive angle and one negative angle that are coterminal with: 255°**

Converting Between Degrees and Radians

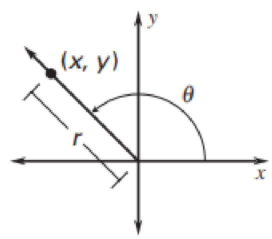
a. Convert −20° to radians. **b.** Convert $\frac{2\pi}{3}$ radians to degrees.

13.3 – Evaluate Trigonometric Functions of Any Angle

VOCABULARY

General Definition of Trigonometric Functions

Let θ be an angle in standard position and (x, y) be any point (except the origin) on the terminal side of θ. The six trigonometric functions of θ are defined as follows.



$\sin \theta = \frac{y}{r}$ $\qquad$ $\csc \theta = \frac{r}{y}, y \neq 0$

$\cos \theta = \frac{x}{r}$ $\qquad$ $\sec \theta = \frac{r}{x}, x \neq 0$

$\tan \theta = \frac{y}{x}, x \neq 0$ $\qquad$ $\cot \theta = \frac{x}{y}, y \neq 0$

Pythagorean theorem gives $r = \sqrt{x^2 + y^2}$.

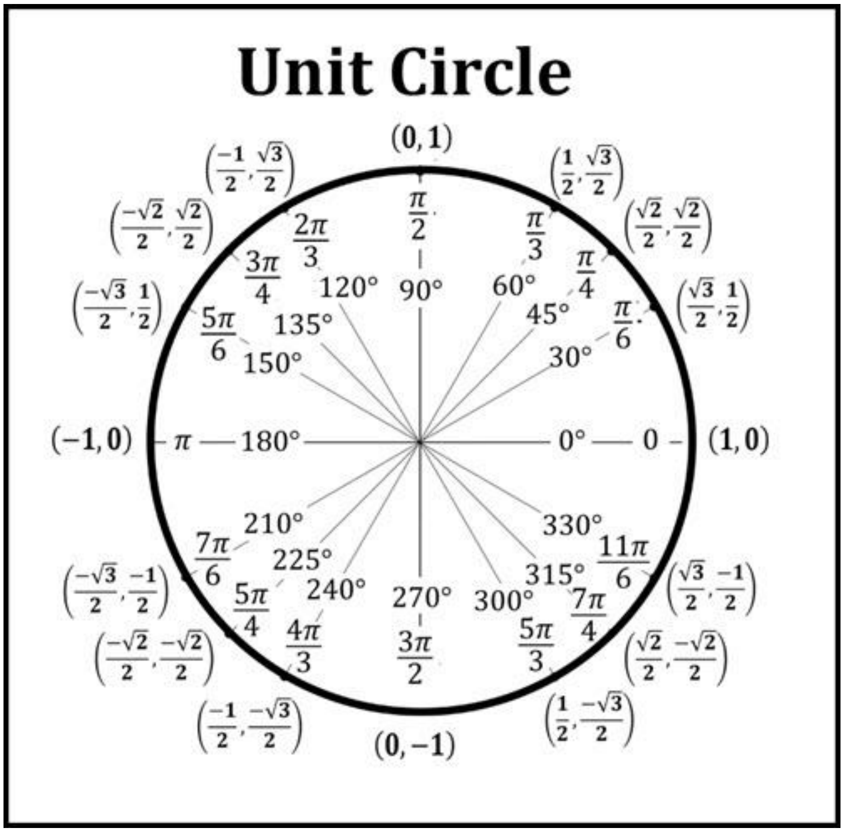
For acute angles, these definitions give the same values as those given by the definitions in Lesson 13.1.

Use the given point on the terminal side of an angle θ in standard position. Then evaluate the six trigonometric functions.

$(-15, -8)$ $\qquad\qquad$ $(1, \sqrt{3})$

- Evaluate the six trigonometric functions of $\theta = \frac{7\pi}{2}$

13.4 – Evaluate Inverse Trigonometric Functions



Evaluate inverse trigonometric functions

VOCABULARY

If $-1 \le a \le 1$, then the **inverse sine** of a is $\sin^{-1} a = \theta$ where $\sin\theta = a$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ (or $-90° \le \theta \le 90°$).

If $-1 \le a \le 1$, then the **inverse cosine** of a is $\cos^{-1} a = \theta$ where $\cos\theta = a$ and $0 \le \theta \le \pi$ (or $0° \le \theta \le 180°$.)

If a is any real number, then the **inverse tangent** of a is $\tan^{-1} a = \theta$ where $\tan\theta = a$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (or $-90° < \theta < 90°$).

Evaluating Inverse Trigonometric Functions

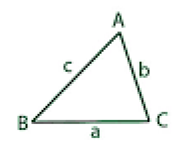
Evaluate the expression in both radians and degrees.

a. $\sin^{-1}(-3)$ **b.** $\tan^{-1}\sqrt{3}$ **c.** $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

13.5 – Apply the Law of Sines

For NON Right Triangles:

AAS, ASA	Law of Sines
SSA	Law of Sines (Ambiguous Case)
SAS, SSS	Law of Cosines

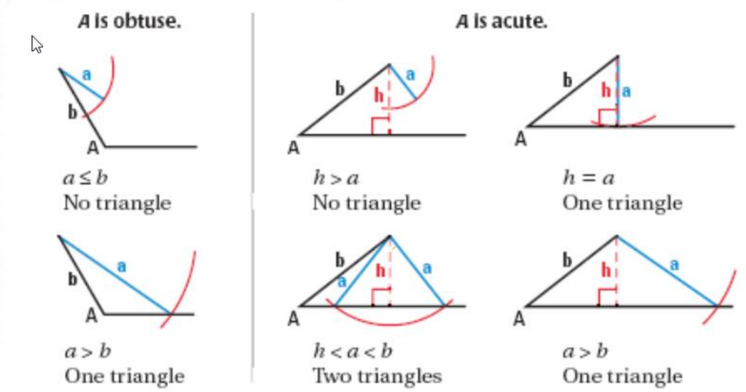


Law of sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Possible Triangles in the SSA Case

Consider a triangle in which you are given a, b, and A. By fixing side b and angle A, you can sketch the possible positions of side a to figure out how many triangles can be formed. In the diagrams below, note that $h = b \sin A$.



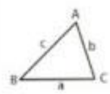
Solve triangle ABC

1. $\boldsymbol{B = 85°, C = 47°, a = 19}$
2. $\boldsymbol{A = 73°, a = 18, b = 11}$
3. $\boldsymbol{C = 98°, c = 29, a = 33}$
4. $\boldsymbol{A = 36°, a = 9, b = 12}$

13.6 – Apply the Law of Cosines

AAS, ASA	Law of Sines
SSA	Law of Sines (Ambiguous Case)
SAS, SSS	Law of Cosines

Law of Cosines



$$c^2 = a^2 + b^2 - 2ab\cos(C)$$
$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

Solve △*ABC*.

1. $A = 62°, b = 56, c = 40$

2. $B = 100°, a = 12, c = 13$

3. $C = 42°, a = 22, b = 35$

4. $A = 56°, b = 15, c = 17$